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PLUME: INSTRUCTIONS FOR USE AND DESCRIPTION OF A MISSILE PLUME VISIBILITY PROGRAM

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NOTICES

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A computer program to demonstrate the ballistics and visual geometry of an air-to-air missile as it is launched against the observer has been developed. This program calculates the probability of visual detection for various mission scenarios as entered by the user. This manual provides detailed instruction for the user and also describes the logic used to develop the program.

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INTRODUCTION

This document is comprised of two main parts and an appendix. The first part describes the uses, and abuses, of the enclosed computer program called **PLUME.** We feel that most users will only need to read this part. The second part is a technical description of the program and gives the reasoning and mathematics used in making the various calculations. Most users will find this second part dull and incomprehensible. We include it, however, in the hope that some future effort can enhance the program and make it even more useful. The appendix is primarily a series of graphs showing the various parameters used in the main program in a more understandable format. This program evolved from a request made in 1986 to the Naval Aerospace Medical Research Laboratory (NAMRL) by the Navy Fighter Weapons School (NFWS) for assistance in determining "the visual detection capability of head-on, forward quarter launched air-to-air threat missile smoke" (1).

Of course, things have evolved somewhat since 1986. When we began work we quickly realized that there could not be any definitive answer to the question posed: that is, how far away will a missile be when the pilot is likely to see it? The answer depends on several variables that must be specified for each case. For example, what is the exact geometry of the interaction? Where is the sun? What is the meteorological range? What type of missile is it? What is the altitude? What terrain background is the missile over? How large is the area the pilot is attempting to cover visually? Assuming a coarse division of all these variables, yields a potential table of "answers" with over 9×10^{19} entries. With 300 entries per page, such a table would require a stack that would reach from Miramar to the moon over 60,000 times!

The Navy has a tradition for generating paper in large quantities but this seemed excessive, even by Pentagon standards. We therefore opted to take a different approach to the problem. After meetings between those of us working at NAMRL and representatives from NFWS, we decided to create a computer program that could provide the needed answers given specific parameters as input to that program. We also expanded the scope of the project to attempt a graphical depiction of what the missile smoke would physically look like after detection, and to allow the operator to maneuver to attempt avoiding the missile.

The resulting computer program is far from perfect. The operator's view is restricted to a small "window" (the computer screen), and spatial orientation is very difficult to maintain. The screen doesn't really look very much like sky, and the missile plume brightness is matched to the screen brightness through a probabilistic manipulation. Still, being "missile bait" on a computer is much more relaxing than the real thing, and ranges of parameters can be explored without serious consequence. (You *must* put up with some snide comments if you should inadvertently find yourself in the same airspace as one of our missiles.) PLUME is about exploring ranges of parameters and "what if" scenarios where the bad guy has live missiles and hostile intentions, but those of us wearing white hats (the good guys) are perfectly safe. Using PLUME to analyze various scenarios may lead to the development of more effective tactics for aircraft deployment.

All the dynamics of the missile guidance and propulsion as well as the target dynamics are as real as could be computer simulated, given realistic limits on the computer's time. The plume diameter expands according to known physical law, and its depiction on the screen is correct for the visual angle used. The missile is limited to a 20-G turn, the target (you) to +7.5 and -2 G. The operator may be surprised to find that he can, if he is alert and cunning, often escape the oncoming missile. A 20-G turn at Mach 3 is, after all, fairly wide compared to a 6-G turn at subsonic speeds.

PLUME models an event that is too dangerous and too expensive to fully explore in the real world -- an oncoming missile. As a model, it lacks the realism of actual events, but is better than nothing at all when it comes to tactical analysis. We hope you find it easy to use and interesting.

SYSTEM REQUIREMENTS

PLUME requires a computer with an 80386 processor (or better) running at a minimum of 25 MHz. The computer must have a hard disk with at least 5 meg of spare capacity. The program is distributed on 5.25-inch, 1.2-meg disks in two versions. One version is compiled for use with a Weitek coprocessor, which is absolutely necessary if a 25 MHz 386 type machine is being used. Another version is compiled for use with an 80387 coprocessor, and may be used with a fast 386 (33 MHz or better) with 80387 coprocessor, or with an 80486 machine running at 25 MHz or better. The 80486 has the xxx87 coprocessor built-in. **PLUME** was developed on a Compaq 386/25 with a Weitek coprocessor. We know it works on this machine, but are only guessing about other machines.

PLUME requires at least 9 meg of main memory. The main sky data arrays have been pre-calculated and are loaded into memory at run time. This speeds up operations considerably; it took a 25-MHz 80486-based machine 2 weeks of continuous running to calculate them.

The target position (you) and maneuvering are controlled with a joystick or a mouse. We used a Gravis Advanced Analog Joystick connected to a Kraft gamecard via the upper port. A Microsoft Mouse should also work for this function, although not as well. The joystick must be calibrated, and we have included a program (JOYSTICK) for doing this.

PLUME will run on DOS 3.3 or later, using the Phar Lap extender to enter protected mode. The Phar Lap extender requirement may be dropped later if DOD should decide to license the run-time program for wider distribution.

GETTING STARTED

PLUME is distributed in compressed form, using the shareware software program PKZIP. We also provide a copy of PKZIP and the companion program PKUNZIP. These are shareware programs, *not* public domain programs. If you find them useful (and we think that you might because they are excellent), follow the directions on the disk and register your copy. The cost of registration isn't much, and the software engineers who wrote these superb programs deserve their due.

The install program will make a directory on the C drive called **PLUME**. You should not already have this directory or there will be a conflict. It will then install the programs into this directory. To start the program, insert disk one into the A drive and type:

C:\>**A:Install**<cr>

where <cr> refers to the carriage return or the Enter key. Continue following screen directions until all four disks are installed.

After installation, change directories to **PLUME** and rename the file **PLUME_WK.EXP** to **PLUME.EXP** if you will be using a machine with a Weitek coprocessor, or rename **PLUME_87.EXP** to **PLUME.EXP** if you will be using the xxx87 coprocessor. To rename, type,

C:\PLUME\> **REN PLUME_WK.EXP PLUME.EXP**

At the prompt, type **PKUNZIP/L <cr>**. This will tell you about licensing this excellent shareware. **PLUME.EXP** is the run-time module, with **bbaray**, **lumaray**, **blaray**, and **lumloray** being data arrays for upper and lower sky luminance's. The "EXP" extension means that **PLUME** is a protected mode program (it can use *all* of the memory) and must be used with the Phar Lap extender called run386.exe. For convenience, we generally rename run386.exe to just run.exe, and in the instructions that follow we will assume that you have also done this with your copy. To do this just type the following at the prompt:

C:\PLUME>**REN RUN386.EXE RUN.EXE** <cr>

The files with the "C" extension are the C language source code files for **PLUME**, those with the "S" extension are in assembly language. **PLUME** is written entirely in C and Assembly, using Microway's NDP C and Phar Lap's Tools. The **CC.DBF** and **AB.LNK** files are used for compiling. Don't play with these files unless you know what you are doing! They are for advanced programmers only, and if you are just going to *use* the program you really don't want to know all the nasty details of why it works. In any case, we are now ready to give it a run.

RUNNING PLUME

Once the files are set up on the computer, running **PLUME** is very easy. We'll take it step by step until you get the basics down. At the prompt, type:

C:\plume>**run plume** <cr>

The computer will spend a moment loading **PLUME** into memory, and then pass control to **PLUME**. The program will then politely ask you to wait while it does some internal calculating and loads the sky data arrays. Don't be lulled into a sense of complacency by this polite demeanor. **PLUME** has the heart of a killer, live missiles, and a definite fondness for pilot meat.

When all is in readiness, the screen will turn to a navy blue color and the basic instructions for using the program will appear. If you read this, you don't really need to read these first two pages and you can just touch any key to proceed. However, you really should read *either* these directions *or* those on the screen.

The first two screens will tell you that **PLUME** was developed at the Naval Aerospace Medical Research Laboratory (**NAMRL**), with a great deal of cooperation and interaction with personnel at the Navy Fighter Weapons School (**TOPGUN**). It models the geometry, ballistics and atmospheric optics of the plume from a missile launched against you. The plume from a missile is a great deal larger and brighter than the missile itself, so we feel certain that it will be the first thing you will see. Just to keep you alert, the launch occurs at a random time after you start the program, from 0-30 seconds. If you try to outmaneuver the computer's missile *before* it's launched, you will

make the machine very mad. This is not a nice computer. It will kill you instantly for doing this. So, it is in your best interest to wait until the missile is launched before taking evasive action.

There will be a small aircraft "icon" at the lower right side of the screen, surrounded by a thick red border. This represents the target (you). You indicate to the computer that you have spotted the plume by flying into the red area as part of your initial escape maneuver. You don't have to do this, of course. You may find that when studying various tactical scenarios it is more convenient to just let the scene play out and give you the data at the end. The data are, of course, why you are doing this. These data include the range when the probability of detection exceeds 75% and the time until impact from this range.

An almost infinite number of scenarios are possible, depending on the specific needs of various missions. You must define the scenario of interest *specifically* in each case, and as a result the output data are *specific*. You have the option of ignoring the visual angle and spot detection entries on the "setup screen" (the third blue screen), and the only result will be that the brightness and visual size of the plume as viewed will not reflect reality to any great degree. The information that appears at the end of each run, giving 75% range of detection and intercept time, will still be accurate for the other parameters entered. However, to use the full capability of the **PLUME** program, you must first match your visual capability in the visual situation you are working in (the room, lighting, screen brightness, etc.). You do this by placing yourself at a distance from the screen that makes the indicated scale bars one degree of visual angle and taking the spot detection test. Once you have these data, assuming conditions don't change, you can just enter the appropriate numbers, place yourself, and proceed.

PLACING YOURSELF FOR PROPER VISUAL ANGLE.

Visual angle refers to the angle, from edge to edge, of a target seen with the eye of the observer taken as the apex of the angle. For example, a four foot wall observed from ten feet would have the same vertical visual angle as a four hundred foot cliff observed from one thousand feet. Both would subtend a visual angle of approximately twenty two degrees. Visual angle is critical when using the **PLUME** program if realism is the goal. If you are using a fourteen inch monitor you should be sitting about five feet back. If you move one foot closer you have changed the visual angle by 20%. When a target is close by, as it is when using the **PLUME** program, small changes in distance have a large effect on visual angle. However, moving several feet when viewing a real missile plume at several miles would not significantly change the visual angle. We present three methods for placing yourself at the appropriate distance from the screen, in decreasing order of accuracy.

Measure the length of the "one degree scale bar" on the screen and calculate the appropriate distance for the eye. This can be done using the following equation:

$$\textbf{Distance From Screen} = \frac{\textbf{Bar Length}}{\textbf{0.01745506}}$$

where **Distance From Screen** and **Bar Length** are in the same units.

Align a "goal post bar" with the one degree scale bar on the screen. A goal post bar can be constructed by placing a tab of metal, one centimeter across, on the end of a stick 57.29 centimeters long. The goal post bar is used by placing the end of the stick away from the tab on the cheek below

the eye and moving forward and backward until the tab just fills the space between the one-degree marks.

A quick rule of thumb is that the width of the thumbnail, held at arm's length, is approximately one degree across. This "thumbnail measure" can be used in place of the goal post bar above by moving back and forth with the arm extended until the width of the thumbnail just fills the space between the one-degree marks. You can "calibrate" your thumbnail with a full moon. The full moon is approximately one half degree in diameter, so two "moons" equals one degree. Howling is optional, but seems to help.

THE SETUP SCREEN

The setup screen is where all the various parameters for a missile shot, your relative position and the position of your wingman are entered. Of course, you may not have a wingman or you may be the wingman so these scenarios can also be indicated. You can move around the setup screen using the arrow keys.

The spot detection test.

The spot detection test is entered by touching the "T" on the keyboard and following the on-screen instructions. This test presents a series of three different sized "plumes" at various intensities in an experimental paradigm known as a "two-interval forced choice with staircase." The staircase is a stepwise incrementing or decrementing of the screen intensity, such that correct answers force the intensity down, and incorrect answers force it up. The time between "beeps" defines two time spaces or intervals. You must chose which interval contained the plume target. The pa digm is "forced" in the sense that you *must* chose one of the intervals to proceed. Most people are surprised to find that when using this type of paradigm they *never* or almost never see the target. Apparently, you can detect things at intensity levels lower than those necessary to let the brain know about. The result, and the purpose, of this test is to allow the computer to adjust the screen brightness to the appropriate level for the probability of detection of a given plume image. Once the probability of detection reaches 99% this brightness limiting is dropped, and the plume becomes fully illuminated so that the form of the plume can be easily seen. The four variables that result from the spot detection test, **i(a)**, **i(b)**, **s(a)**, and **s(b)**, should be noted to avoid the necessity of retaking the test. If a printout of the results of a run is made, these will be included for your convenience.

Increasing the plume brightness.

You may wish to increase the plume brightness initially in order to observe the program in action more clearly. This is easily accomplished by arbitrarily increasing the value of the spot detection variables. Try increasing **i(a)** to 15.

Scan area.

The scan area refers to the area you must view to search for incoming missiles. If this area is large, the probability of detection goes down. Scan area is entered in degrees horizontal and vertical. If the absolute maximum detection range must be found, a scan area of one degree, vertical and horizontal, could be used. The most sensitive area of the human eye, the fovea, is approximately

two degrees in diameter so making your scan area less than this will not accomplish anything. The program assumes a random scan paradigm with a fixation change (a saccade) every one-third second.

Meteorological Range.

The meteorological range is a measure of the "clearness" of the atmosphere, and is often called the "visibility." A meteorological range of forty nautical miles would mean that a large object, such as a mountain, was just visible at that distance. Meteorological range does not consider the effects on visual angle of increasing distance.

Terrain

Terrain type can make a difference in the visibility of a missile plume if the geometry of the interaction is such that the terrain forms a background to the plume. There are fourteen terrain types programmed into **PLUME**. They are listed in figure one, below, or you can see a list by touching the question mark (?) key. After viewing the list, note the number associated with the terrain type of interest and enter it in the space indicated.

TERRAIN BACKGROUNDS

Terrain background data is taken from Table 3.2 of J.I. Gordon's 'Optical Properties of Objects and Backgrounds,' APPLIED OPTICS Vol. 3, No. 5, 1964.

The data are incomplete and have been extrapolated to fill gaps. The most complete data sets were 1 and 5; use others with caution. Pick the terrain of interest, note the number and return to the main program to enter it.

1. **Pine trees, small, uniformly spaced (these are best data, from Eglin AFB).**
2. **Grass, thick, long, pale green, dormant, dryish, little ground showing.**
3. **Asphalt, oily, with dust film blown onto oil.**
4. **'White' concrete, aged.**
5. **Calm water, infinite optical depth (very clear).**
6. **Grass, lush green, closely mowed thick lawn.**
7. **Macadam, washed off and scrubbed.**
8. **Dirt, hard packed, yellowish.**
9. **Mixed green forest, deciduous (oak) and evergreen (pine).**
10. **Pine forest (these data and 9 above from near Julian, CA in 1959).**
11. **Grass, dry meadow, dense, midsummer.**
12. **Ilyas, sparse and dry, yellowish grass on sand at end of summer.**
13. **Sand dunes, sharply expressed micro-relief, dry.**
14. **Podsol, ploughed, moist.**

<touch any key to continue>

Figure one. Terrain background screen.

Sun bearing.

The position of the sun relative to the scenario is very important to the results. If the plume is close to the sun, glare will render it almost impossible to detect. If the sun is behind you, the plume will be nicely illuminated and probably quite visible. Other positions have intermediate effects. You must enter the sun bearing relative to your direction of flight. For example, if the sun is directly off your port side, it would be at 270 degrees. The sun altitude (height above the horizon) is fixed at 41.5 degrees because this is the only position for which we had good data.

Missile type.

We have included two "generic" missiles, one with a single-burn motor and one with a dual-burn motor. The program can accommodate four more missile types to be entered by the user (by a programmer experienced with the "C" programming language). You select the missile type by highlighting it using the arrow keys. Both the generic missiles have a "main beam avoidance maneuver" where they drop approximately 100 feet at launch. This maneuver requires about two seconds to complete. The missiles use proportional navigation, aiming at an anticipated intercept point. The capture "cone" angle for the seeker head is set up in a header file and is eighty degrees in diameter as the program is delivered. If you find yourself escaping with little effort you may have set the scenario up so that the missile never acquires the target. Realize that G limits are placed on the maximum missile turn rate with low gain far from the target and high gain close to the target. All of the missile parameters were standardized to a comprehensive series of computer simulations (see the appendix).

The battle scenario.

There are three players in the battle scenario, a **target** (usually you), a **spotter** (always you), and a **shooter**. All directions entered at the setup screen are taken relative to the target, so the data entry for the target is simply his altitude and speed.

The spotter is effectively "linked" to the target and could be thought of as a wingman. The perspective seen on the screen is the perspective of the spotter. If the spotter is a distance off the target's starboard side and a missile is incoming from directly ahead of the target, then the spotter will see the missile plume streaming from left to right. The wingman has a seeming advantage over the target here, as his perspective causes the visual angle of the missile plume to be larger, thus presumably causing it to be easier to detect. Try it and see for yourself; you may be surprised! In some cases, an increase in visual size caused by the change in perspective is more than offset by a decrease in plume contrast (it is "thinner"). Two things contribute strongly to visual detection, the area over which search must be made and the contrast of the target being searched for. There is a spatial summation effect on contrast. Spatial summation refers to the fact that as a visual target gets larger it can be dimmer and remain at the threshold of detection. However, spatial summation only holds up to a point, that point being roughly twenty eight square arc-minutes of visual area. Beyond this point, increases in size do *not* appreciably increase spatial summation and, therefore, do not increase effective contrast. If a wingman has to increase his visual scan area to look for targets fired at *his* wingman, this may reduce the overall probability of detection. When scan fields are large the increase in the area of the scan field decreases the probability of placing a particular visual fixation on the plume.

The spotter can be "placed" almost anywhere relative to the target. If his distance from the target is 0.0 then the spotter and the target are the same: there is no wingman. If the spotter is placed at 045 degrees and 0.6 nautical miles at the same altitude, then the spotter would be seen by the target ahead and to the right. No speed entry is required for the spotter: it is the same as the target.

The shooter is also placed relative to the target, with the assumption that he is flying directly toward the target in order to bring his missiles to bear. If he is at 000 (relative to the target), he would be straight ahead on a collision course! With closing velocities being the vector sum of both the target and the shooter, things can happen rapidly with this scenario. The shooter's altitude, distance, speed, and relative bearing are whatever you want them to be; just remember that they are relative to the target.

After setting it all up.

After setting up all the parameters for a particular missile shot, press the spacebar twice to run the scenario. You will then see a brief screen that says,

Use the joystick to fly in the clear sky area until you spot the plume.
AFTER you spot it, fly into the red area to evade!

The forward button throttles up, the rearward one down.

<<<< The spacebar will stop and start the program >>>>

These instructions refer to a joystick type of control. A mouse may also be used, in which case the left button throttles down and the right button up. Next, a screen will appear similar to figure two, below.

Figure two has several salient features that should be noted. The instruments along the bottom-left reflect the status of the **target** as designated in the setup screen. In this particular case, the target is traveling at Mach 0.8. The target's original heading is straight ahead (this is always true; all bearings are relative to the target, so his heading is, by definition, at 000) but will probably change as he maneuvers. The target is level at 20,000 feet of altitude, and he is not pulling any G.

Along the left side and top of the screen are two scale bars with red pointers. The scales are in degrees of visual angle, relative to the original heading of the target, and indicate the position of the missile if it is flying, or the position the missile will appear from if it is not. When we were originally testing this program we always had the missile oriented in the center of the screen, with the screen tracking as if a "window" were following the missile flight. However, it proved extremely difficult for the operator to maintain a sense of spatial orientation with this arrangement. The scale bars help somewhat, but do not be surprised if you are a bit confused about the spatial arrangement of all entities the first few times you try the program. The problem is not readily solvable, because the computer screen itself is only so large.

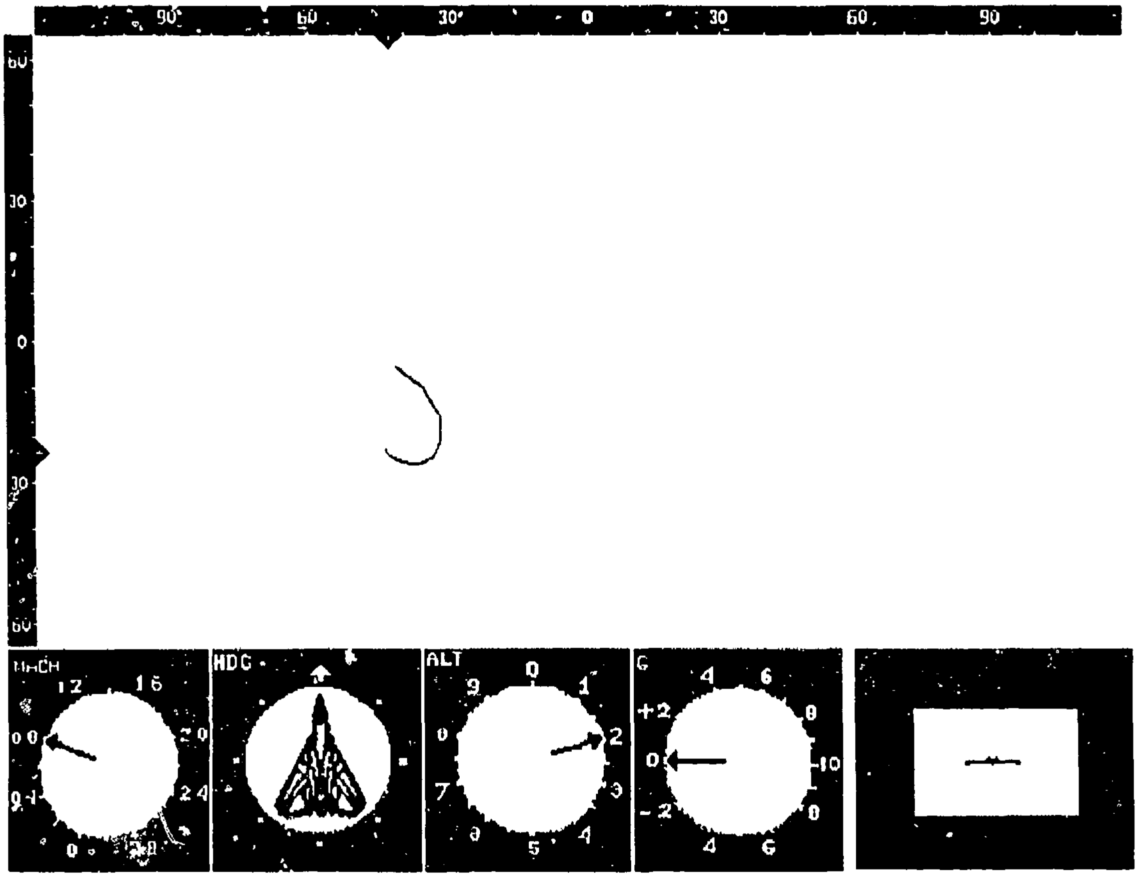

Figure two. The PLUME screen.

You can pause the action at any time by pressing the space bar. Pressing again causes the action to resume.

A box at the lower right side of the screen with thick, red borders contains an airplane icon which will respond to the joystick. The operator can "fly" the target-spotter pair with the joystick and see relative movement with the airplane icon. The red border provides the computer with a means of "knowing" when the operator has spotted the plume. You indicate this by "flying" into the red area (presumably as part of an escape maneuver) as soon as the "tally ho" is made. However, we realize that nobody likes to be "killed" by a computer and that there would be a strong temptation to maneuver *before* the tally ho was actually made--for whatever reason. To keep the operator honest, the computer will instantly kill any operator that performs such an early maneuver!

Finally, a word about the plume itself. Figure two does not present a very good representation of a plume; we drew this one in by hand to give you some idea what to look for. The physical dimensions of the plume depicted during a real computer run should, however, be fairly accurate. We base the plume dimensions on known physical laws governing gas expansion at various altitudes, et cetera. The *brightness* of the plume is not accurate. We manipulate the brightness of the plume in an attempt to match the probability of detection we calculate for the scenario (which is based on known detection data) with the probability of detection of your seeing the particular pattern on the screen you are using in whatever conditions you are operating under. Which is why the "spot detection" test is important. However, we have placed a "switch" in the logic that says that when the probability of detection exceeds 99% the brightness is incremented up to max in order to

improve your ability to discern the plume shape. We are not overly happy with this approach, but it was the best we could do given the equipment we have to work with and what we anticipate are the needs of the user.

After your narrow escape.

You *did* escape, didn't you? In either case, the next screen (after the snide remarks if you found yourself in the same airspace with a missile) shows the relevant data for **that particular** scenario. These data may resemble those shown in figure three below.

The top three lines of data indicate basic range and time data for a given probability level. The probability level is not exactly 75% because probabilities change significantly three times a second as eye fixations occur, and these probabilities will rarely fall exactly on 75%. The range, intercept time, and real-world probability of detection at time of first sighting plus reaction time indicate data captured at the instant when you flew the icon into the red area. The missile range at the point of closest approach is just what it says. When this range is forty feet or less, it is considered a hit (proximity fuse in missile) and you die. When you get to this screen touch the 'P' key to print your data and save the results from the vision test. Other messages may appear as the situation warrants. For example, if you fly into the ground the computer will tell you about it (after making you crash and burn), but this section completes what you should know to run the program. The next section is an explanation of the technical approach taken in producing the **PLUME** program.

Range when probability of detection exceeds 75% (in nm) -------> 0.8
Exact probability level at range indicated above --------------------> .89
Missile intercept time (seconds) at prob. level above --------------> 1.6

Range at time of first sighting + reaction time (in nm) -----------> 0.7

Time between time of first sighting + reaction time and
calculated missile intercept (in seconds) ----------------------------> 1.1

Calculated real-world probability of detection at time of
first sighting + reaction time ---> .98

Missile range at point of closest approach (in feet) ----------------> 1.7

<Touch 'P' to print data, spacebar twice to try it again>

Figure three. Data screen.

TECHNICAL DETAILS

The sections that follow present the logic used in writing the **PLUME** program. Each section is fairly general, intending to show an approach to the problem more than the exact solution. If an exact solution is required, the program source code should be consulted. The source code is written in the 'C' programming language with the more demanding sections (demanding in the sense of time dependent) in assembly language.

The main module of **PLUME** is called **PLUME.C.** Two headers called **SCREEN.H** and **MISSILE.H** contain several variables that can be changed by the user for recompilation. For example, as shipped, the aluminum oxide content of the plume exhaust is set at one percent via a header variable called **AL_OXIDE**. There are four 'C' language modules that are called from **PLUME.C**; they are: **SCREEN.C**, which handles many of the details involved in the video part of the program; **DYNAMICS.C**, which contains the routines for calculating the dynamics of the various entities; **LOGO.C**, which handles the details of the screen-user interface; and **SYNOPSIS.C**, which handles the final data output screens and print utilities. In addition, there are two assembly language modules: **VGA.S**, which is a collection of routines used by the program, many of which are concerned with the vga screen; and **ANI.S**, which contains the assembly language routines for keeping track of pixels that have been written and their location, so that redundant writes to the screen or erases are avoided. The system **BIOS** (Basic Input Output System) has been avoided for all time-intensive operations (such as screen writes) to keep the speed of operation as fast as possible. Instead, the **VGA** video card has been directly addressed.

Detection probability; an overview.

The probability of detection of a particular visual event involves three main factors. First, the physical characteristics of the target in question must be calculated. A missile plume is a product of burning material spewing hot gasses into an atmosphere at a particular rate, at a particular altitude, and with a particular composition. Missile propellants with a high aluminium oxide content are particularly visible. Before any further calculations can be made, the nature of the plume itself, and most particularly the geometry of its appearance from the point of view of the observer, must be known. Second, the target must have sufficient contrast to become visible. Color contrast tends to wash out when large distances are involved, so the absolute value of the contrast is paramount. To calculate visibility in **PLUME,** we first calculate the visual size and reflectivity of the plume, integrate the light falling upon it from the particular direction it is observed, factor in the glare effect of the sun (if appropriate), calculate the contrast relative to the background, and then reduce this contrast as a function of the amount and type of atmosphere the image must pass through. This contrast, as seen by the observer, is then compared to Blackwell's (2) contrast-detection data to determine if the threshold for detection has been exceeded. The assumption is then made that the target has a 100% probability of detection if the observer looks directly at it. Third, the probability of the observer orienting his eye on or near the object of interest is calculated. If eye orientation is directly placed on the object of interest (the image of the object falls on the fovea), then detection at maximum range is possible. Off center orientations will reduce detection range. Normal human observers cannot produce smooth eye movements when scaning a large visual field; they move their eyes in a series of fixations (called saccades). *During* the saccade, vision is suppressed. Saccades typically occur three times a second when scanning a visual area. Consequently, the larger the area to be scanned the lower the probability of spotting a particular small target.

Detection probability; some details.

Mathematical Model of Proportional Missile Navigation.

The missiles in **PLUME** use proportional navigation to seek their target. In other words, they calculate an intercept point and aim for that point rather than for the target itself. Proportional navigation is depicted in figure four.

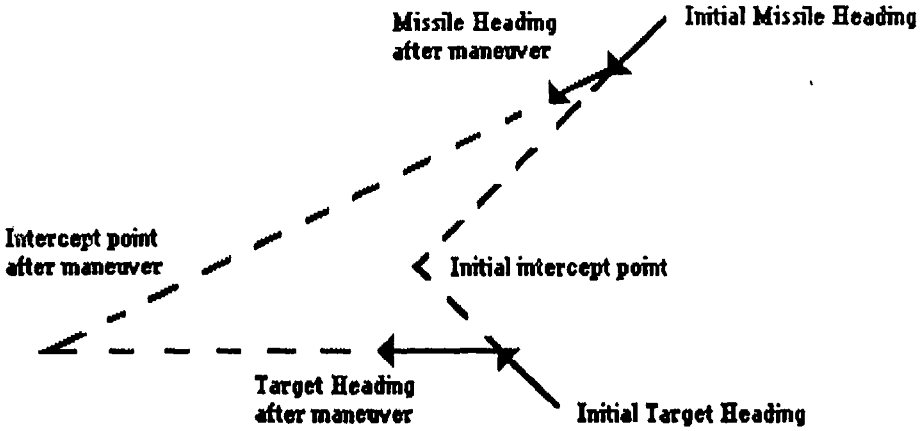

Figure four. Missile navigation.

The initial intercept heading is set at the time of launch. It is based on the shooting aircraft's launch parameters and assumes that the steering cues for the shot are centered. During the **main beam avoidance maneuver** the missile will drop approximately 100 feet while accelerating. The initial course is held for two seconds, but after this time the missile can maneuver for intercept. The greatly increased speed of the missile, combined with maintenance of the initial course, may require the missile to execute a large initial turn. However, during minimum range shots the seeker head may lose the target because of gimbal limitations. The main beam avoidance maneuver is very advantageous to the target because it may cause the missile to lay a great deal of plume across the sky in a way that aids the observer in detection.

The linear acceleration of the missile is based on a simplified model of the forces acting on it; these are thrust, gravity and drag. Transition points are included in the thrust parameter (high boost, low boost, burnout). Drag is computed using velocity and atmospheric density at altitude. Missile mass is reduced during motor burn in proportion to the amount of efflued matter. All of these parameters were matched to a comprehensive series of tests conducted at the Pacific Missile Test Center.

Mathematical model of plume gas generation.

PLUME uses an exhaust gas generation model to compute the properties of the plume. This model is based on the work of Libby, as modified by Victor and Breil (3). In essence, it calculates the plume diameter and particle density which contains 90% of the gas flow at any point in space during the flight of the missile. Using the formulae in reference 3, this analysis was condensed to the solution of a 4th degree equation for fc, the mass fraction of jet gas on centerline of the plume

at position x. This equation may be written as follows.

$$0 = 30.25\ fc^4 + 0.1551\ fc^3 + C_1 fc^2 - C_2 fc - Te\ \mu e\ B$$

where: $C_1 \quad = 0.00019881 - \frac{B}{2}(Tj - Te)(\mu j - \mu e)$

$C_2 \quad = B\left[(Tj - Te)\ \mu e + \frac{Te}{2}(\mu j - \mu e)\right]$

$B \quad = \frac{144\ \mu j\ rj^2}{(\mu j - \mu e)^2 X^2\ Tj}$

fc = mass fraction of jet gas on centerline of plume at position x.
X = longitudinal position of plume.
Te = freestream temperature (°K).
Tj = plume centerline starting temperature (°K).
μj = plume gas starting mean velocity (m/sec).
μe = free stream velocity (m/sec).
rj = initial radius of the plume, = $0.02\sqrt{\frac{F}{Pe}}$ where
F = thrust (Kilonewtons).
Pe = atmospheric pressure (atm).

Using fc, the plume diameter, d, can be derived.

$$d = 2.8\ rj\left[\frac{Pj}{Pc}\frac{\mu j}{\mu_{1/2}}\right]^{1/2} Y_{1/2}$$

Where: $\frac{Pj}{Pc} = \frac{Tc}{Tj}$

Pj = starting point pressure.
Pc = centerline pressure.
Tc = fc(Tj - Te) + Te
$\mu_{1/2} = \frac{\mu e - \mu c}{2}$
$\mu c = fc(\mu j - \mu e) + \mu e$
$Y_{1/2} = 5.5 + \frac{0.0141}{fc}$ for fc less than or equal to 0.1.
$Y_{1/2} = 1.49 + \frac{0.105}{fc} - \frac{0.000513}{fc^2}$ for fc greater than 0.1.

A comprehensive review of variables affecting the plume may be found in the appendix, along with many plots showing graphical analysis.

PLUME VISIBILITY

Plume visibility is a function of plume brightness relative to the brightness of the background (usually the sky) as it appears to the observer (contrast) and target size. The target size is a matter of geometry, with size being expressed in units of visual angle. Plume contrast involves several factors, which are discussed below.

Illumination from the sky.

See references 3, 4, and 5 for the source of the following formulae.

Sky illumination was calculated using the following relationship:

$$\mathrm{Sky}_{\mathrm{illum}} = \left(\frac{\lambda}{2\pi}\right)^2 \int_{\mathrm{allsky}} \frac{\Sigma n\,(i_1+i_2)}{2} I_{sky}\, d\, \phi\, D_X$$

where:
- λ = the wavelength of incident light, = 550 nm.
- Σn = 2.5×10^{12} particles per meter^3 (uniform plume assumption from reference 3).
- $i_1 + i_2$ = the mie function for a given viewing angle and particle size.
- I_{sky} = the illumination of the sky at a given altitude, zenith and azimuth angle.
- d = the diameter of the plume in meters.
- ϕ = the weight percent of aluminium oxide in the propellant times 1.89.
- D_x = dilution of particle concentration = $0.5 \left(\frac{r_j}{r_{1/2}}\right)$
- r_j = radius of plume in meters.
- $r_{1/2}$ = radial portion in plume at half-velocity radius of plume.

Sky luminance data as a function of azimuth of the sun, altitude and zenith angle were taken from Boileau's data (6) and interpolated to produce data tables with increments of two degrees vertically and horizontally around the sphere. The double integral was then calculated using Simpson's rule of integration for double integrals. The resulting interpolation and integration tables used in the **PLUME** program are named **BBARAY** for the upper sky interpolated data; **LUMARAY** for the integrated upper sky data; **BLARAY** for the interpolated lower sky data; and **LUMLORAY** for the integrated lower sky data. These arrays must be present on disk for **PLUME** to load at run time. In designing the **PLUME** program we were constantly seeking to optimize the running speed of the program. To do this we precalculated everything we could and loaded the results into arrays. The program is quite large because of this, but memory is cheaper than fast processors. As a point of interest, it took a 486 computer running continuously at 25 MHz over two weeks to calculate the above arrays. The programs that produced the arrays above are included with the source code and are called **UPPERSKY.C, LUM_SKY.C, LOW_SKY.C,** and **LUML_SKY.C,** respectively.

MIE scattering is defined as scattering produced by spherical particles without special regard to comparative size of radiation wavelength and particle diameter, which is a good approximation of the random particle size and random wavelengths to be found in a missile plume. The mie function for a given viewing angle, θ, (complement of angle relative to a particular piece of

sky, plume, and viewer) was obtained from a curve fit of graphical data from Victor and Breil (3) as follows.

$$i_1+i_2 = \begin{cases} 0.000533185\,\theta^2 - 0.0737622\,\theta + 3.133 & \text{for } \theta < 76^0 \\ -0.00009224386\,\theta^2 + 0.003305\,\theta + 0.8709967 & \text{for } 76^0 \leq \theta < 114^0 \\ 0.004042425\,\theta - 0.390194 & \text{for } 114^0 \leq \theta < 146^0 \end{cases}$$

$$i_1+i_2 = \begin{cases} 0.0008613326\,\theta^2 - 0.2340955\,\theta + 16.01778 & \text{for } 146^0 \leq \theta < 169^0 \\ -0.001833748\,\theta^2 + 0.6714747\,\theta - 60.05204 & \text{for } 169^0 \leq \theta < 180^0 \end{cases}$$

The equations are shown in expanded form to aid the interested reader in ascertaining their derivation from the original sources.

Illumination from the sun.

The effect of sun illumination on the plume was calculated as follows:

$$Sun_{illum} = \left[\frac{\lambda}{2\pi}\right]^2 \frac{\Sigma n\,(i_1+i_2)}{2} E_{sun}\, d\, \phi\, D_X$$

where λ, d, θ, D_x, and Σ_n are the same as defined above.
$i_1 + i_2$ = the mie function for viewing angle θ (sun, plume, viewer).
E_{sun} = the solar illuminance (dependent on altitude and zenith angle).

We derive the value for E_{sun} as follows (6).

$$E_{sun} = I_s\,(1 - a\ Shooter_{alt})\,E^{(\sec z)^s} + a\ Shooter_{alt}\, I_s$$

where a = 0.14.
s = 0.678.
Is = $2.0 \cdot 10^9 \cdot 8.895 \cdot 10^{-5}$
$E = e^{-u}\left[1 - \frac{u}{3}\right]$.
$u = \frac{3280.839895}{4\,a\,V_r}$.
V_r = the meteorological range in nautical miles.
$Shooter_{alt}$ = the shooter's altitude in kilometers.

Plume transmittance.

Plume transmittance is approximated as follows (3).

$$Plume_{trans} \cong [\,e^{-V_i\, d\, \phi\, D_x}\,] - 1$$

where V_i = -0.93.
d, ϕ, and D_x are as defined above.

Calculation of plume brightness.

Plume brightness is calculated from the simple sum of three components. First, the sunlight scattered back to the observer's eye by the plume; second, the illumination from the sky itself (plus the ground) scattered back to the observer's eye; and third, the light that passes through the plume from the background.

$$\text{Plume}_{\text{brightness}} = \text{Sun}_{\text{illum}} + \text{Sky}_{\text{illum}} + \text{Plume}_{\text{trans}}$$

This quantity represents the brightness of the plume *at the plume* and not necessarily at the observer. As light from the plume travels to the observer through a "tunnel" of air some of it is scattered out of the "tunnel" causing the brightness to be degraded. Light is also scattered into the tunnel, but this light averages the same as light scattered into the area around the "tunnel" so the effect on contrast is to again reduce it.

Calculation of apparent plume and background brightness.

Two separate formulae are used for computing the apparent plume brightness and apparent brightness of the background, depending on the relative altitudes of the observer and plume.

Case one, the plume is higher than the observer.

$$\text{Plume}_{\text{app\_brightness}} = \text{Plume}_{\text{brightness}} \left(\frac{\text{Observer}_{\text{trans}}}{\text{Plume}_{\text{trans}}} \right) + \text{Path}_{\text{lum}}$$

and $\text{Background}_{\text{app brightness}} = \text{sky}_{\text{illum}}$ at plume altitude.

where $\text{Path}_{\text{lum}} = \text{sky}_{\text{illum}}$ at observer altitude.

$$\text{Observer}_{\text{trans}} = e^{-\left[\frac{z \sec \theta}{L_z}\right]}$$

$$\text{Plume}_{\text{trans}} = e^{-\left[\frac{z \sec \theta}{L_z}\right]}$$

z = altitude.

θ = inclination of the path of sight.

L_z = the equivalent attenuation length as defined by Boileau (6).

Case two, the observer is higher than the plume.

$$\text{Plume}_{\text{app\_brightness}} = \text{Plume}_{\text{brightness}} \left(\frac{\text{Observer}_{\text{trans}}}{\text{Plume}_{\text{trans}}} \right) + \text{Path}_{\text{lum}}$$

$\text{Observer}_{\text{trans}}$ and $\text{Plume}_{\text{trans}}$ as in case one above.

Background$_{app\_brightness}$ = inherent luminance of the background due to different types of terrain, times the beam transmittance at the plume altitude, plus the sky brightness at the observer's altitude.

Path$_{lum}$ = **sky**$_{illum}$ at observer minus **sky**$_{illum}$ at plume position times ratio of transmittances at observer and plume.

Inherent luminance of background = I_{sun} times a terrain factor interpolated from measured data (8).

Case three, the observer and the plume at the same altitude.

The case of the observer and the plume at the same altitude is treated as the limit of cases one and two.

Calculation of plume contrast.

Plume contrast against its background is calculated using the following formula.

$$\text{Plume}_{contrast} = \left| \frac{\text{Plume}_{app\_brightness} - \text{Background}_{app\_brightness}}{\text{Background}_{app\_brightness} + \text{Glare}} \right| .$$

$$\text{where} \quad \text{Glare} = \left[\begin{array}{c} 29.0 \cdot 8.89578 \cdot 10^{-5} \left[\dfrac{E_S}{(\theta + 0.13)^{2.8}} \right] \text{ for } \theta \leq 3.3^{o} \\ 10.0 \cdot 8.8578 \cdot 10^{-5} \left[\dfrac{E_S}{\theta} \right] \text{ for } \theta > 3.3^{o} \end{array} \right]$$

θ = the angle between the sun plume and observer.
E_s is defined above.

Threshold contrast assuming a foveated target.

Once the contrast of the target is determined, the assumption can be made that an observer *will* see it if the contrast exceeds some threshold value and is presented in the observer's direct line of sight. The question is, what is the criterion threshold level? For the answer, we can use Blackwell's data (2), but these are inconveniently presented in tabular form. To address this issue, we made an analytic fit to Blackwell's data (2). The expression describing this fit is as follows:

$$\text{Threshold}_{contrast} = 0.006 + \left[\frac{0.3}{\text{area}^{0.75}} \right] \left[\frac{1.0 + 0.62 - 0.135 \log_{10} \text{area}}{\text{Background}^{0.33}} \right]^{3}$$

where **area** = the target area in $arcmin^2$.
Background = is the background luminance in foot Lamberts.

Probability of detection as a function of area to be scanned.

The point when the target luminance exceeds the threshold value for foveal detection defines a maximum range for detection, but does not necessarily reflect a realistic situation. For this range to be accurate, the observer would have to be looking directly at the missile plume. A more realistic depiction of missile-plume-detection range must take into account the high probability that the observer's eye will *not* be directed toward the plume, at least not at precisely the time when the plume exceeds the minimum threshold for detection. The human eye is most sensitive on its visual axis, with sensitivity dropping off quickly with eccentricity. This effect is depicted graphically in figure five, below.

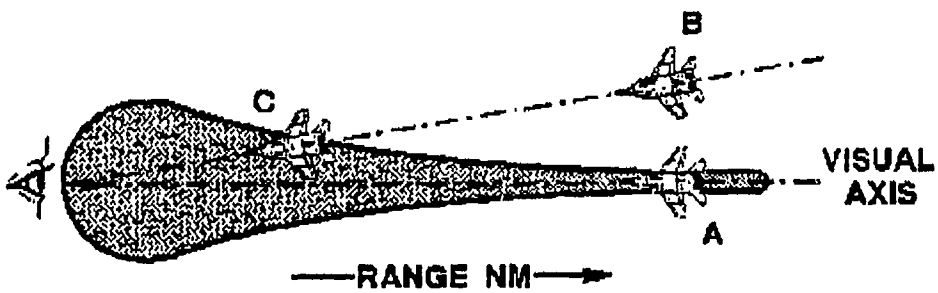

Figure five. The visual detection lobe.

Airplanes A and C in figure five are visible to the observer, but airplane **B** is not. If the observer in figure five was interested in detecting airplane **B** and made an eye movement that "skipped over" the target, his saccadic suppression (the visual suppression during rapid eye movements) might prevent him from detecting the target *even though it was on his visual axis at some point.* Visual search strategies that can concentrate the area of a series of fixations will improve the probability of detection of a small target. The **PLUME** program assumes that fixations are randomly distributed over the area being searched, and occur three times a second.

An algorithm for calculating the probability of detection of a given target, given the target size (in arcmins of visual angle) and the area to be scanned (in degrees of visual angle) has been developed by Mr. James Harris of Harris Visibility Studies in San Diego, California (private communication). That algorithm is described below.

Calculating the probability of orienting the visual axis on a target given a specific scan area.

The visual detection lobe shown in figure five depicts the "edges" of a "just detectable threshold" as a function of eccentricity from the visual axis. The distance to the edge of the visual lobe from the visual axis, in degrees, can be calculated with the following formula:

$$Lobe_{radius} = \frac{\left[\dfrac{0.25 \cdot Plume_{app\_contrast}}{\left[\dfrac{0.24}{area} + \dfrac{0.025}{\sqrt{area}} + 0.0075\right]}\right] - 1}{0.8}$$

Where $Lobe_{radius}$ = the visual detection lobe radius in degrees.
$Plume_{app\_contrast}$ = the apparent contrast of the target.
area = the area of the target in squared arcmins.

The solid angle associated with this radius is calculated as follows:

$$Solid_{lobe} = \pi \cdot (Lobe_{radius})^2$$

There will be some inefficiency in the search pattern near the edges. In order to account for this, an "overlap" equal to the diameter of the visual detection lobe is built into the "Field of View" calculation.

$$Field_{of\_view} = [Horiz_{field} + 2 \cdot Lobe_{radius}] \cdot [Vert_{field} + 2 \cdot Lobe_{radius}]$$

Where $Field_{of\ view}$ = the scan area in degrees squared.
$Horiz_{field}$ = the horizontal scan field, in degrees.
$Vert_{field}$ = the vertical scan field, in degrees.

The probability of a "glimpse" is then calculated:

$$P_{glimpse} = \frac{Solid_{lobe}}{Field_{of\_view}}$$

The probability of *not* detecting the plume is then obtained by subtraction:

$$P_{no\_detect} = 1 - P_{glimpse}$$

But glimpses, or fixations, only occur three times a second. A cumulative probability of no detection can now be calculated as follows:

$$PCum_{no\_detect} = \prod_{i=1}^{n} P_{no\_detect}$$

And finally, the cumulative probability of detection may be calculated:

$$PCum_{detect} = 1 - PCum_{no\_detect}$$

Once the probability of detection for the plume has been calculated (every 1/3 second), the screen brightness can be matched to this probability via the probability of detection for similar images determined with the built-in "spot detection" test.

The appendix contains program listings for three programs written in BASIC for the Macintosh computer. Two of these programs were used to analyze the linear balistics of a missile and the plume generation. The third program is a graphical analysis package. Several graphs showing the results of analysis are reproduced.

Appendix:
Ballistic and Rocket Motor Plume Analysis
of
Air-to Air Missiles

by
Steven Schallhorn

This appendix contains three BASIC programs, written for the Apple Macintosh computer, for analyzing the linear ballistics and plume generation of an air-to-air missile, along with the graphical depiction of these analyses. Several examples of analysis are also included.

The **Linear B .llistic** model computes the velocity and distance traveled by the missile. It is based on simple physical principles implemented with an integration routine. Thrust is computed as a function of time during propellant transition periods. The missile mass is computed as a function of fractional loss of propellant. The model uses real-time analysis of drag and gravitational forces as well as changes in atmospheric conditions.

The **Plume Generation** model computes the diameter of the rocket motor plume enveloping 90% of the exhaust particulate matter for primary smoke at a given down-range distance. It is based on the work of P.A. Libby as modified by A.C. Victor and S.H. Breil, Naval Weapons Center, China Lake, CA (3). The program is intended to be used with the Linear Ballistic model. It assumes unity Prandl and Schmidt numbers as well as representative values for various rocket motor chamber terms. A Newtonian approximation routine solves for the mass fraction of jet effluent in the exhaust plume centerline(**Fc**). This solution was designed to be most accurate when values of **Fc** are less than 0.1. **Fc** values of interest here are generally less than 0.01. The half velocity radius of the plume is computed as well as the diameter.

The **Graphic Analysis** is a series of computations on the Linear Ballistic and Plume Generation models displayed with a graphic format. Several important variables are explored as to their effect on the missile ballistic and plume characteristics. Comparison is made between prototypical "single" (boost only) and "dual" (boost and sustain) thrust missiles. Motor thrust and burn times were created so that the missiles have the same specific impulse.

Definition of Terms

	Term	Sample value
Ta	Total elapsed time (sec)	Ta = 15
Mach	Launch velocity (mach)	Mach = 0.9
Altitude	Launch altitude (ft)	Altitude = 20000
Angle	Missile dive angle (deg)	Angle = 0
Thrust	"DUAL" or "SINGLE"	Thrust$ = "DUAL"
tb1	Boost motor burn time (sec)	tb1 = 3.2
tb2	End of sustain burn (sec)	tb2 = 13.4 (tb1 = tb2 for "SINGLE" thrust)
FV1	Boost motor thrust (lbs)	FV1 = 6000
FV2	Sustain motor thrust (lbs)	FV2 = 1000 (FV2 = 0 for "SINGLE" thrust)
Mass	Initial missile weight (lbs)	Mass = 500
Propellant	Propellant weight (lbs)	Propellant = 100
Uj	Initial plume gas mean velocity (m/sec)	Uj = 3000
Tj	Initial plume gas mean temperature (deg K)	Tj = 2000
Vo	Initial velocity (kts TAS)	Vo = Mach*(661-0.0022*Altitude)
mp	Propellant mass (slugs)	mp = Propellant/32.17
Te	Temperature at altitude (deg K)	Te = 288.15-0.0019812*h
Pe	Pressure at altitude (atm)	Pe = (288.15/Te)^(-5.255877)
Cd	Drag coefficient	Cd = 0.00001
g	Term for atmospheric density	g = 5654.3*Pe^1.18/Te
stp1	Time interval	stp1 = 0.1
stp2	Integration interval	stp2 = 0.01
w	Normalizes mass to thrust	w = mp/(FV1*tb1+0.3*(FV1-FV2)+FV2*(tb2-tb1-0.3))

Variables

d	Diameter (ft) of plume at distance S and time Ta
f	Instantaneous thrust of missile
Fc	Mass fraction of gaseous jet effluent in exhaust plume centerline at distance S and time Ta
h	Altitude (ft) of missile at time t
m	Instantaneous missile mass (slugs = wt/32.17)
p	Ratio of centerline temperature to initial temperature
R	Half velocity radius normalized to rj
rj	Initial plume radius (ft)
s	Distance (nm) traveled by missile at time t
Uc	Centerline plume velocity (m/sec)
vel	Velocity (kts) of missile at time t
x	Imaginary distance traveled by a constant velocity missile at time Ta

Linear Ballistic Model

Basic Language for the Macintosh Computer

REMEMBER: This program generates the linear velocity and down-range distance traveled by an air-to-air missile. Thrust is computed as a function of time during .6 second transition periods. The mass is then computed as a function of fractional loss of propellent. It uses real-time analysis of drag and gravitational forces as well as changes in atmospheric conditions.

```
FOR t=0 TO Ta STEP stp1

IF t=0 THEN vel=Vo : f=FV1 : s=0 : GOTO NEXT t  'Sets initial values

FOR T1=stp2 TO stp1 STEP stp2
```

REMEMBER: T1 is a subroutine for the integration of vel and s and assumes that the integration interval (stp2) is shorter than the plume generation interval (stp1).

```
IF t<=tb1 THEN f=FV1                        'Computes thrust as a function of time
IF t>tb1 AND t<=tb1+.6 THEN f=(FV2-FV1)/.6*(t-stp1+T1-tb1)+FV1
IF Thrust$="SINGLE" THEN GOTO 30
IF t>tb1+.6 AND t<=tb2 THEN f=FV2
IF t>tb2 AND t<tb2+.6 THEN f=FV2*(1+(tb2-t+stp1-T1)/.6)

30 IF t>=tb2+.6 THEN f=0
m=m-w*stp2*f                                'Computes mass as a function thrust
Drag=Cd*g*(6080/3600)^2*(vel)^2             'Drag in terms of velocity and atmospheric density
Gravity=m*32.17*SIN(angle*3.14159/180)      'Gravitational effects for a given dive angle
Force=f+Gravity-Drag                        'Sums forces acting on missile

vel=vel+3600/6080*Force/m*stp2              'Integration for missile velocity (kts)
s=s+vel/3600*stp2                           'Integration for missile down-range distance (nm)

h=h-6080/3600*vel*stp2*SIN(angle*3.14159/180)'Re-computes altitude for a given dive angle
Te=288.15-.0019812*h                        'Re-computes temperature at new altitude
Pe=(288.15/Te)^(-5.255877)                  'Re-computes pressure at new altitude
g=5654.3*Pe^1.18/Te                         'Re-computes density term at new altitude

NEXT T1
```

REMEMBER: PLUME GENERATION MODEL BELONGS HERE

```
NEXT t
```

Plume Generation Model

Basic Language for the Macintosh Computer

REMEMBER: This program computes the plume characteristics at a distance s and time Ta. This plume was first created at time t with a missile velocity of vel. The linear ballistic model is used to generate these values.

```
rj=.00437*(f/Pe)^.5                              'rj=initial plume radius (ft) at time t
Ue=vel*.5145 : rj=rj/3.28                        'Converts values to metric
a=Tj-Te : b=Uj-Ue
x=(Ta-t)*Ue
```

REMEMBER: x=imaginary distance traveled by a missile (using the same velocity as when the plume was first created) at elapse time Ta

```
IF x=0 THEN d=2*rj : GOTO NEXT t

c=144*Uj*rj^2/(b^2*x^2*Tj)                       'Subroutine solution of Fc
c1=.00019881#-a*b*c/2                            'Solution most accurate with Fc<.1
c2=(a*Ue+b*Te/2)*c
IF Ta-t<=.1 THEN STOP                            'Prevents endless loop
IF Ta-t<.5 THEN Fc=.1 ELSE Fc=.01                'Sets initial Fc value
y=30.25*Fc^4+.1551*Fc^3+c1*Fc^2-c2*Fc-Te*Ue*c
45 y1=121*Fc^3+.4653*Fc^2+2*c1*Fc-c2             'Newtonian solution for Fc
Fc=Fc-y/y1
```

REMEMBER: Fc=Mass fraction of gaseous jet effluent in exhaust plume at distance S and time Ta

```
y=30.25*Fc^4+.1551*Fc^3+c1*Fc^2-c2*Fc-Te*Ue*c
IF ABS(y)>1E-11 THEN GOTO 45                     'Establishes accuracy of estimation

Uc=Fc*(Uj-Ue)+Ue                                 'Uc=Centerline plume velocity (m/sec)
U=.5*(Ue+Uc)                                     'U=Half velocity
p=1/Tj*(Fc*(Tj-Te)+Te)                           'p=Ratio of centerline temp to initial temp
Xa=.3/Fc
y2=5.5+.047*Xa
IF Fc>.1 THEN y2=1.49+.35*Xa-.0057*Xa^2          'Approximates values for Fc>.1
R=(p*Uj/U)^.5*y2                                 'R=Half velocity radius normalized to rj

d=2.8*R*rj*3.28                                  'd=Diameter at distance S and elapse time Ta
```

REMEMBER: By gaussian distribution, the diameter of the plume is 2.8X the half velocity radius

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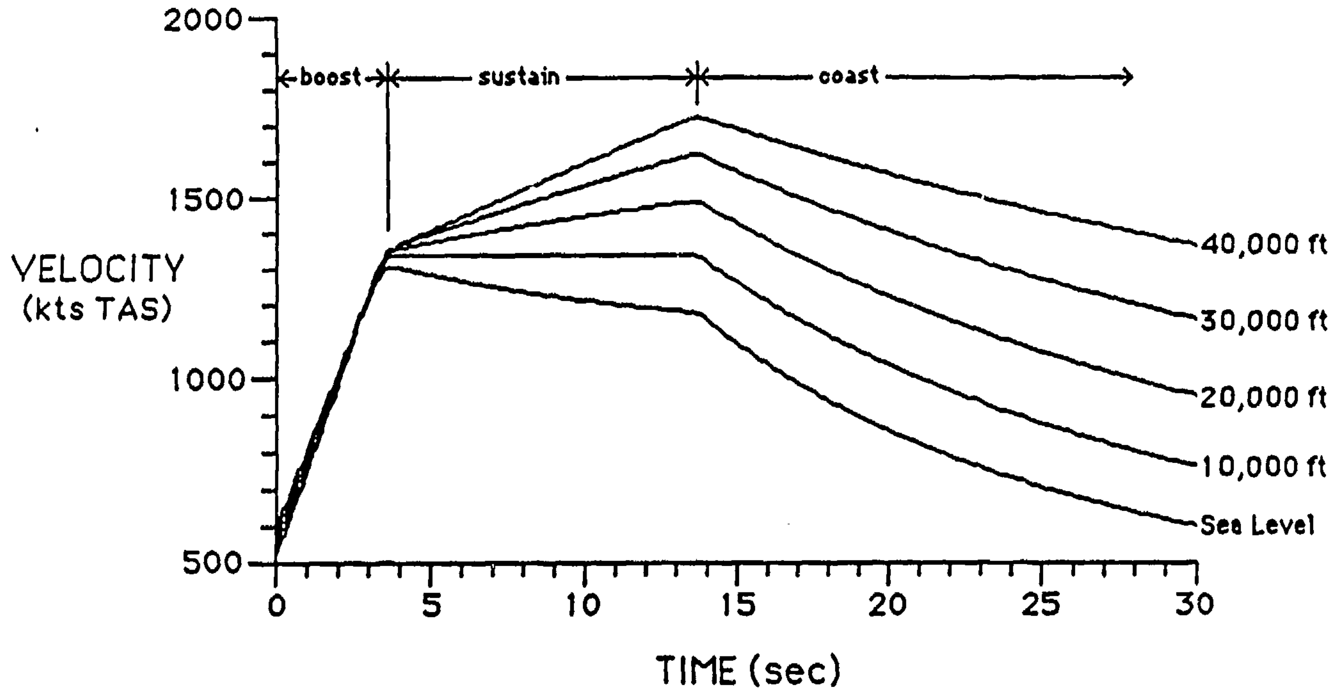
VELOCITY VERSUS TIME
with VARIATION in LAUNCH ALTITUDE

Dual thrust missile
3.2/13.4 sec burn times
Level flight

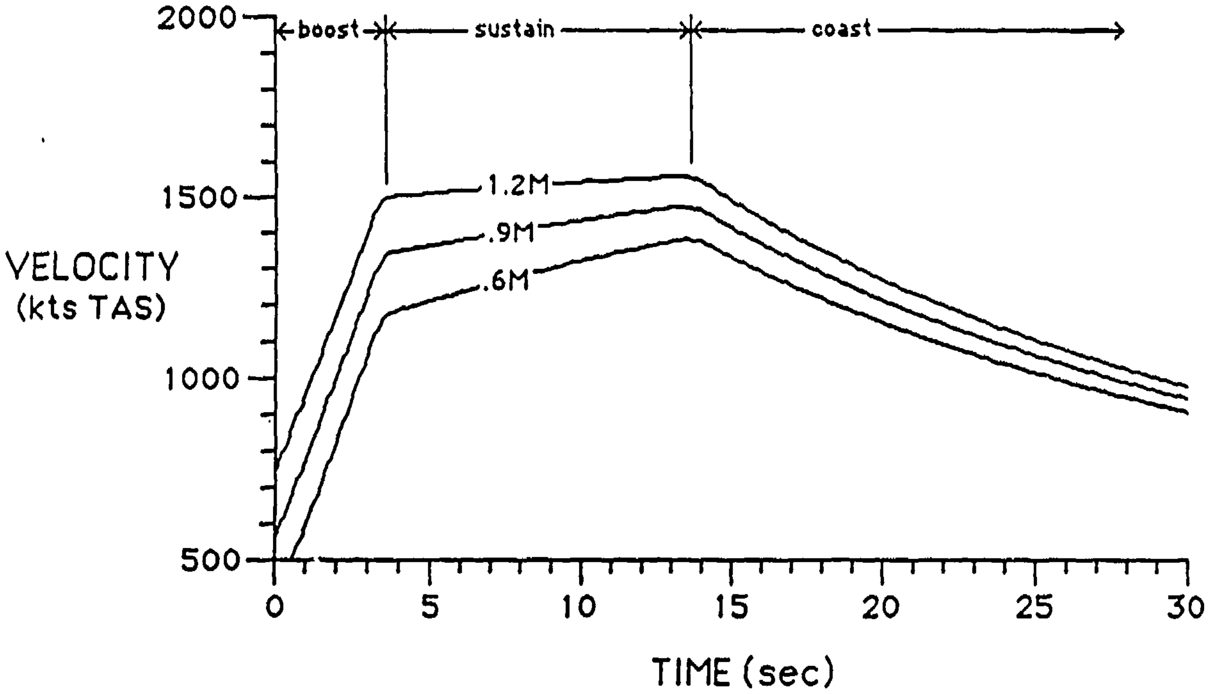


Launch Velocity

Sea Level: .9M = 595 kts TAS
10,000 ft: .9M = 575 kts TAS
20,000 ft: .9M = 555 kts TAS
30,000 ft: .9M = 535 kts TAS
40,000 ft: .9M = 516 kts TAS

VELOCITY VERSUS TIME with VARIATION in LAUNCH VELOCITY

Dual thrust missile
20,000 ft launch altitude
Level flight

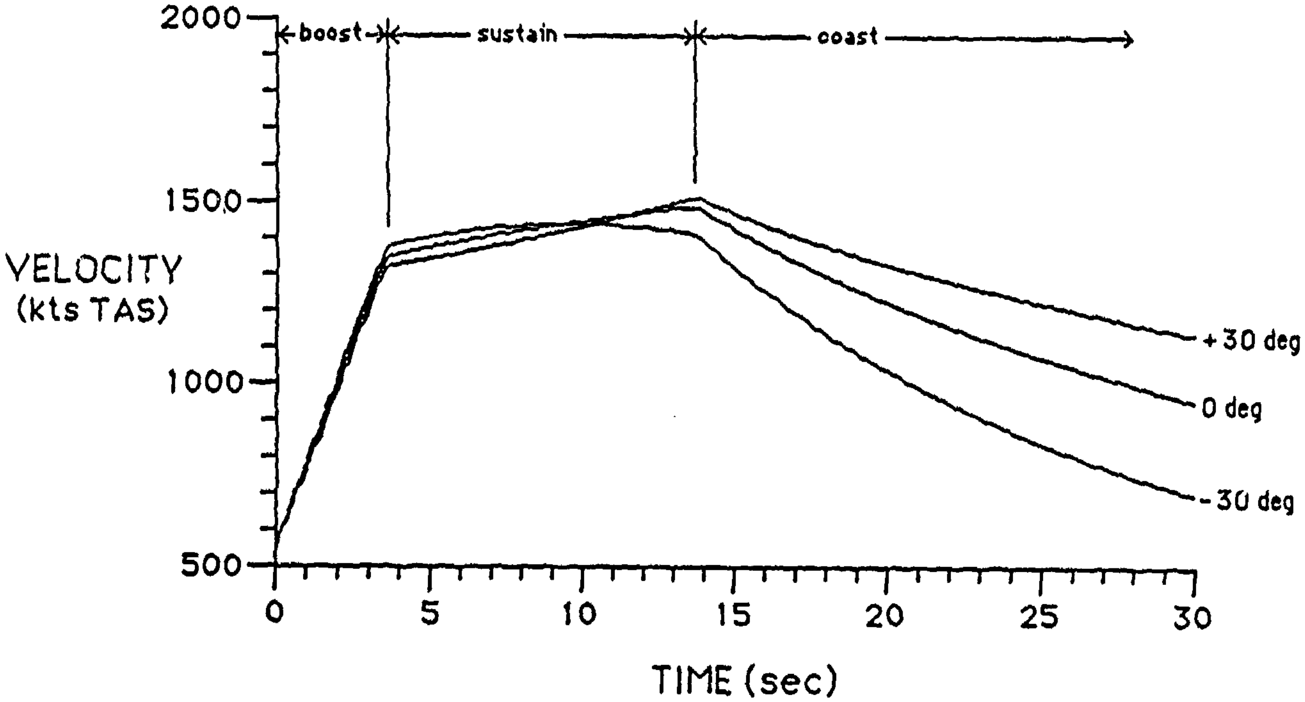


Launch Velocity

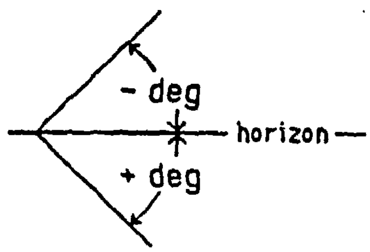
.6M = 370 kts TAS
.9M = 555 kts TAS
1.2M = 740 kts TAS

VELOCITY VERSUS TIME with VARIATION in DIVE ANGLE

Dual thrust missile
20,000 ft launch altitude
.9M launch velocity

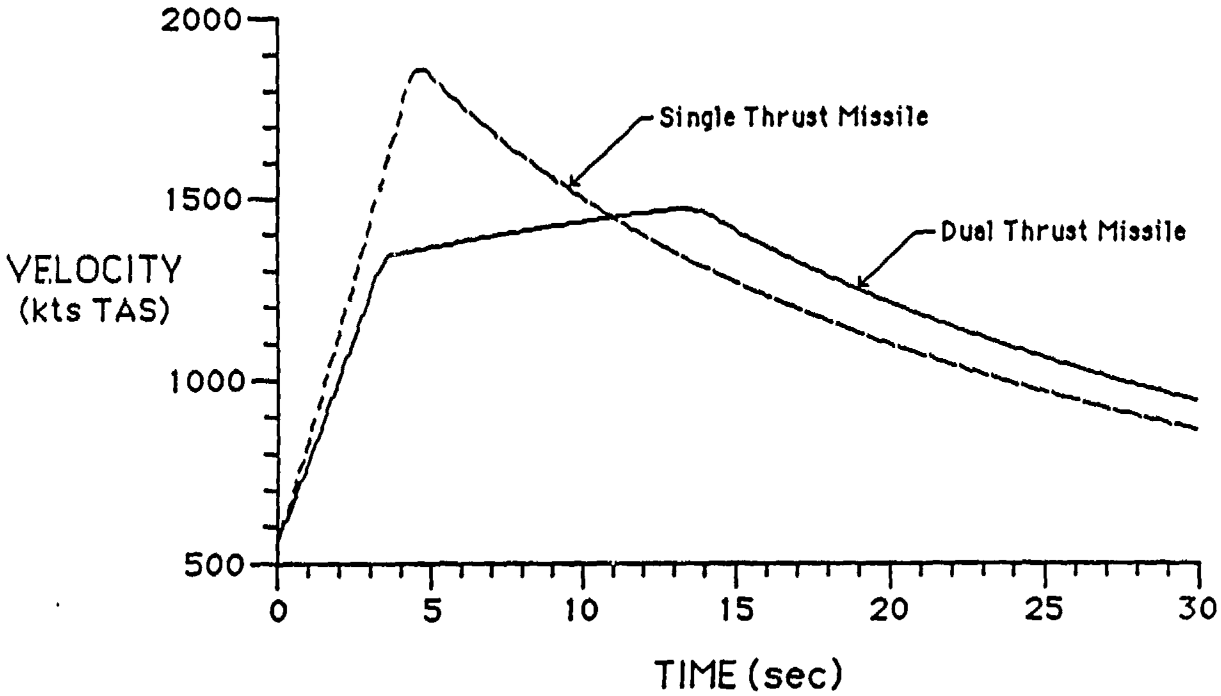


Dive Angle	Altitude (20 sec)
-30 deg	35,300 ft
0 deg	20,000
+30 deg	4,520



VELOCITY VERSUS TIME COMPARISON BETWEEN "SINGLE" and "DUAL" THRUST MISSILES

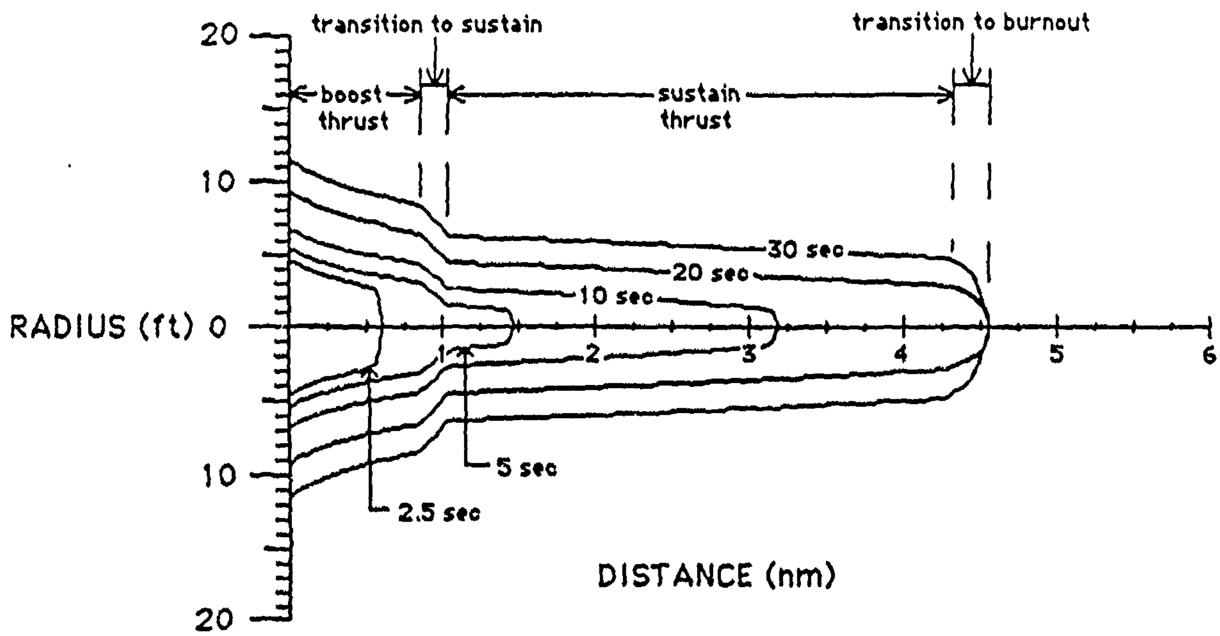
20,000 ft launch altitude
.9M launch velocity
Level flight



Missile	Burn Time(s)	Thrust
Single	4.4 sec	7500 lb
Dual	3.4/13.4	6000/1000

DOWN-RANGE PLUME RADII for VARIOUS ELAPSE TIMES

Sea level launch altitude
Level flight



Average Plume Diameter:

Elapse time	Boost dia.	Sustain dia.
2.5 sec	6.77 ft	-
5.0	8.12	2.87 ft
10	10.76	4.11
20	15.22	7.60
30	19.30	11.42

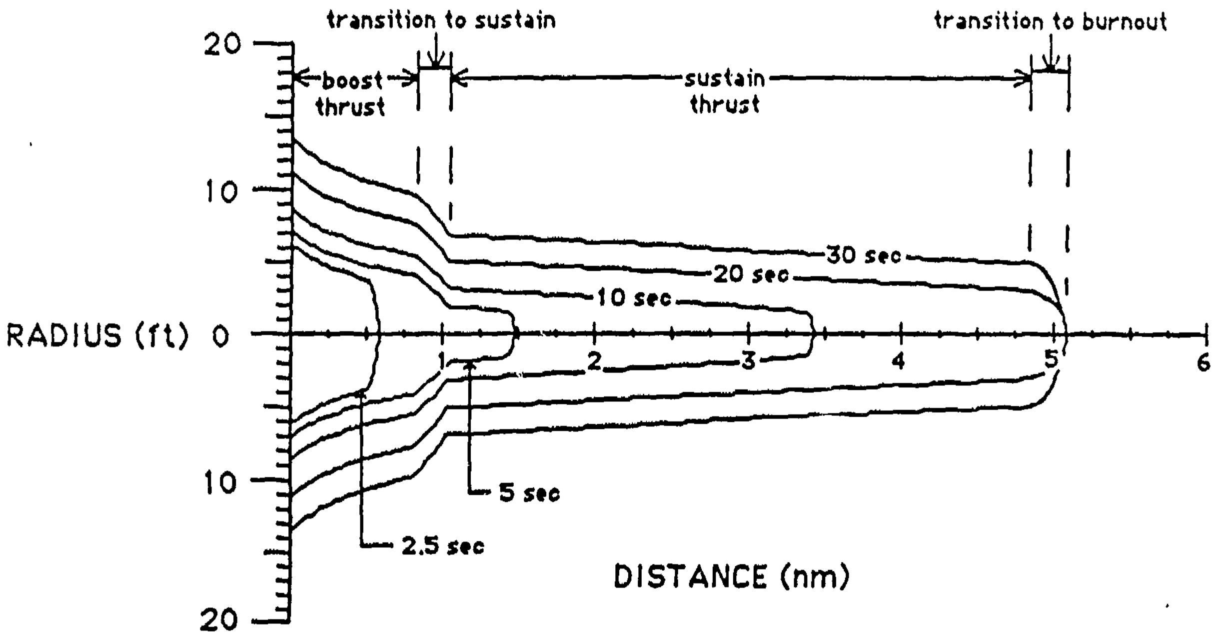
Dual Thrust Missile Parameters:

Thrust	Time of flight	Distance
Boost motor (6000lb)	0-3.2sec	0-.83nm
Transition to sustain	3.2-3.8	.83-1.04
Sustain motor (1000)	3.8-13.4	1.04-4.34
Transition to burnout	13.4-14.0	4.34-4.53

Launch velocity: .9M (595kts TAS)
Initial plume mean temperature: 2000deg K
Initial plume mean velocity: 3000m/sec
Initial missile weight: 500lb

DOWN-RANGE PLUME RADII for VARIOUS ELAPSE TIMES

20,000 ft launch altitude
Level flight



Average Plume Diameter:

Elapse time	Boost dia.	Sustain dia.
2.5 sec	8.95 ft	-
5.0	10.44	3.67 ft
10	13.29	4.80
20	17.99	8.24
30	22.30	11.90

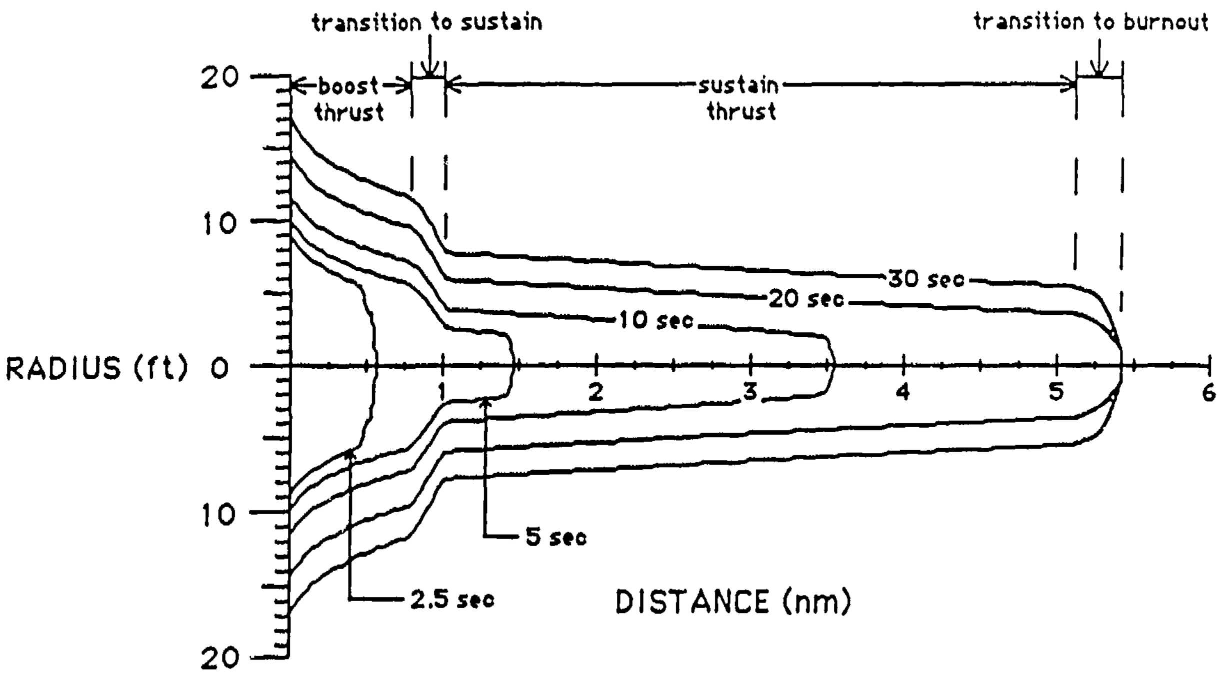
Dual Thrust Missile Parameters:

Thrust	Time of flight	Distance
Boost motor (6000lb)	0-3.2sec	0-.81nm
Transition to sustain	3.2-3.8	.81-1.03
Sustain motor (1000)	3.8-13.4	1.03-4.84
Transition to burnout	13.4-14.0	4.84-5.09

Launch velocity: .9M (555kts TAS)
Initial plume mean temperature: 2000deg K
Initial plume mean velocity: 3000m/sec
Initial missile weight: 500lb

DOWN-RANGE PLUME RADII for VARIOUS ELAPSE TIMES

40,000 ft launch altitude
Level flight



Average Plume Diameter:

Elapse time	Boost dia.	Sustain dia.
2.5 sec	12.59 ft	-
5.0	14.34	5.07 ft
10	17.49	6.11
20	22.58	9.57
30	27.09	13.28

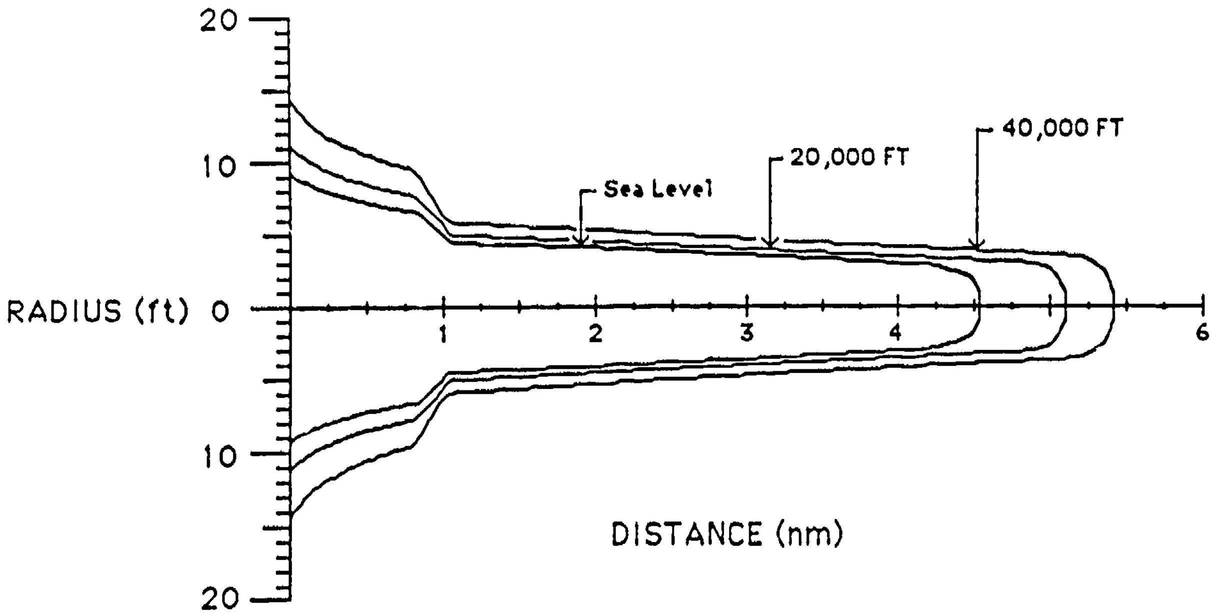
Dual Thrust Missile Parameters:

Thrust	Time of flight	Distance
Boost motor (6000)	0-3.2sec	0-.79nm
Transition to sustain	3.2-3.8	.79-1.01
Sustain motor (1000)	3.8-13.4	1.01-5.13
Transition to burnout	13.4-14.0	5.13-5.42

Launch velocity: .9M (516kts TAS)
Initial plume mean temperature: 2000deg K
Initial plume mean velocity: 3000m/sec
Initial missile weight: 500lb

DOWN-RANGE PLUME RADII for VARIOUS LAUNCH ALTITUDES

20 sec elapse time
Level flight

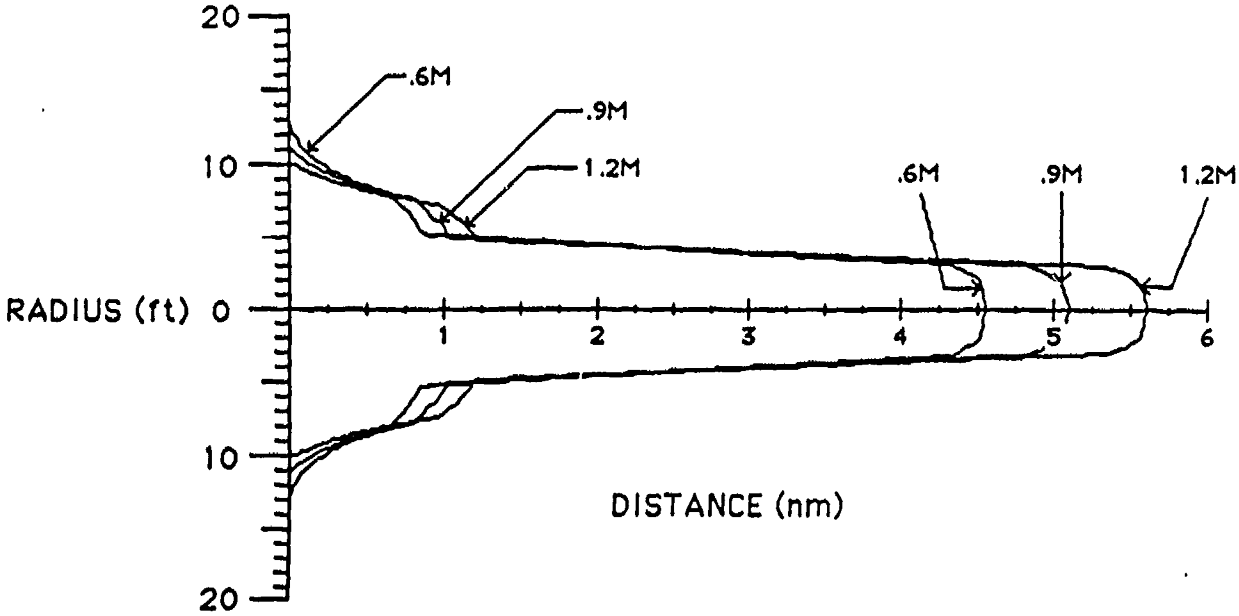


Altitude	Boost dia.	Sustain dia.	Plume length	Launch Velocity
Sea Level	15.22 ft	7.60 ft	4.53 nm	.9M = 595kts TAS
20,000 ft	17.99	8.24	5.09	.9M = 555kts TAS
40,000	22.58	9.57	5.42	.9M = 516kts TAS

Dual thrust missile with 3.2/13.4 sec burn times
Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII for VARIOUS LAUNCH VELOCITIES

20,000 ft launch altitude
20 sec elapse time
Level flight

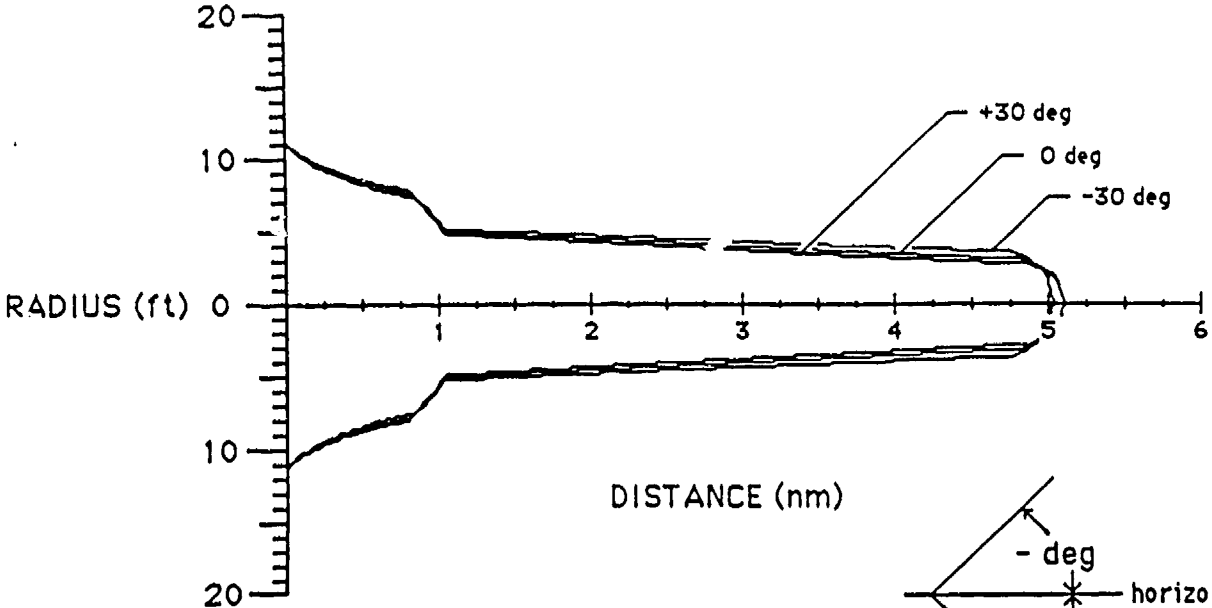


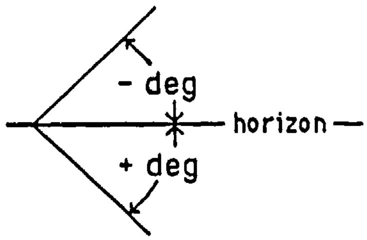
Launch Velocity	Boost dia.	Sustain dia.	Plume length
.6M = 370 kts TAS	19.29 ft	8.49 ft	4.58 nm
.9M = 555 kts TAS	17.99	8.21	5.09
1.2M= 740 kts TAS	16.92	7.95	5.61

Dual thrust missile with 3.2/13.4 sec burn times
Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII for VARIOUS DIVE ANGLES

20,000 ft launch altitude
.9M launch velocity
20 sec elapse time



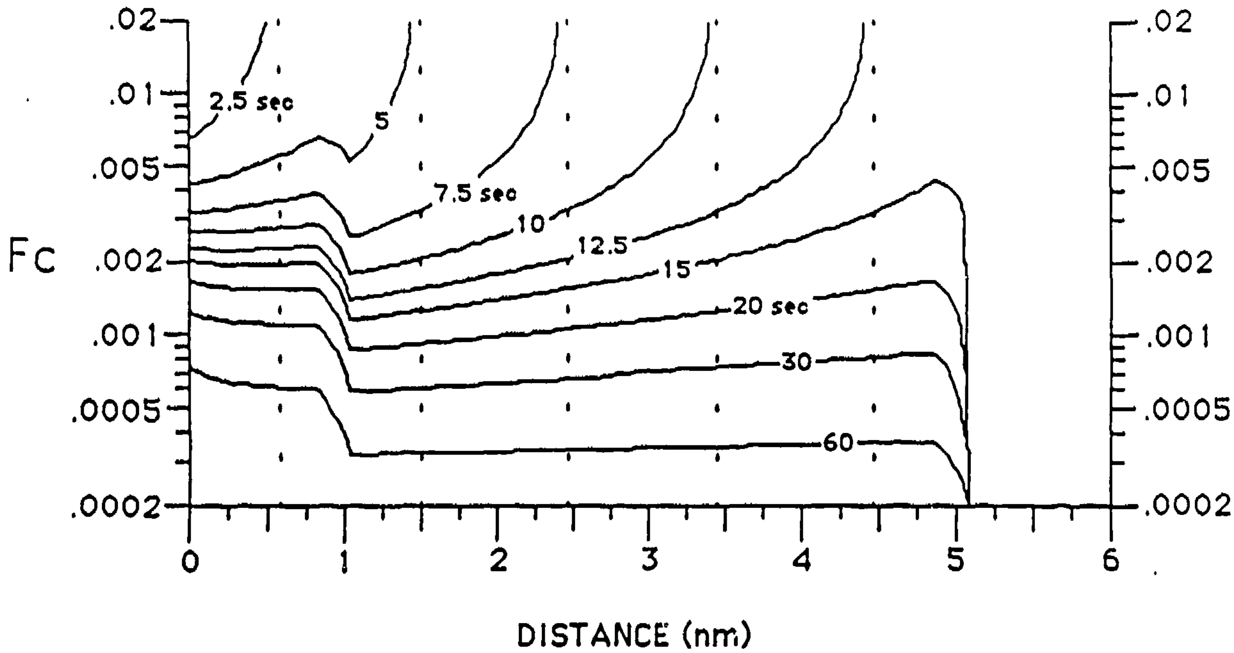


Dive Angle	Boost dia.	Sustain dia.	Plume length	Final Altitude
-30 deg	18.29 ft	8.88 ft	5.03 nm	35,300 ft
0 deg	17.99	8.21	5.09	20,000
+30 deg	17.68	7.70	5.09	4,520

Dual thrust missile with 3.2/13.4 sec burn times
Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

Fc VERSUS DISTANCE for VARIOUS ELAPSE TIMES

20,000 ft launch altitude
.9M launch velocity
Level flight



Dual Thrust Missile Parameters:

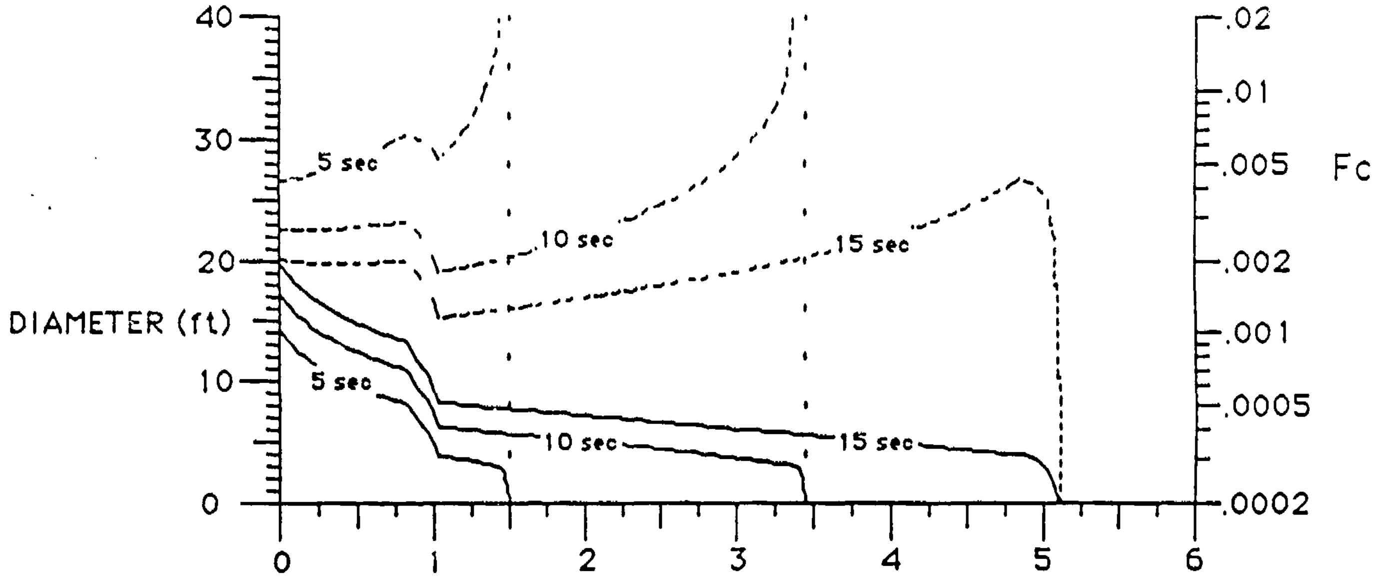
Thrust	Time of flight	Distance
Boost motor (6000lb)	0-3.2sec	0-.81nm
Transition to sustain	3.2-3.8	.81-1.03
Sustain motor (1000)	3.8-13.4	1.03-4.84
Transition to burnout	13.4-14.0	4.84-5.09

Launch velocity: .9M (555kts TAS)
Initial plume mean temperature: 2000deg K
Initial plume mean velocity: 3000m/sec
Initial missile weight: 500lb

Fc represents the mass fraction of gaseous jet effluent in the plume centerline

PLUME DIAMETER and Fc VERSUS DISTANCE for VARIOUS ELAPSE TIMES

20,000 ft launch altitude
.9M launch velocity
Level flight



Dual Thrust Missile Parameters:

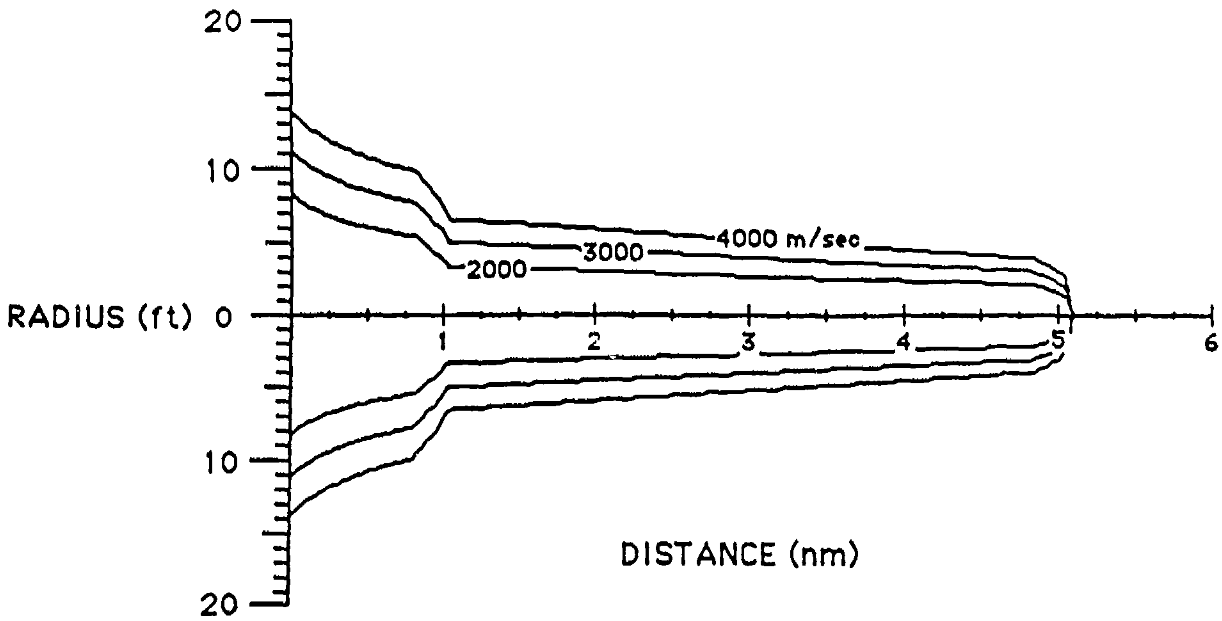
Thrust	Time of flight	Distance
Boost motor (6000lb)	0-3.2sec	0-.81nm
Transition to sustain	3.2-3.8	.81-1.03
Sustain motor (1000)	3.8-13.4	1.03-4.84
Transition to burnout	13.4-14.0	4.84-5.09

Launch velocity: .9M (555kts TAS)
Initial plume mean temperature: 2000deg K
Initial plume mean velocity: 3000m/sec
Initial missile weight: 500lb

Fc represents the mass fraction of gaseous jet effluent in the plume centerline

DOWN-RANGE PLUME RADII for VARIOUS INITIAL PLUME CENTERLINE VELOCITIES

20,000 ft launch altitude
20 sec elapse time
Level flight

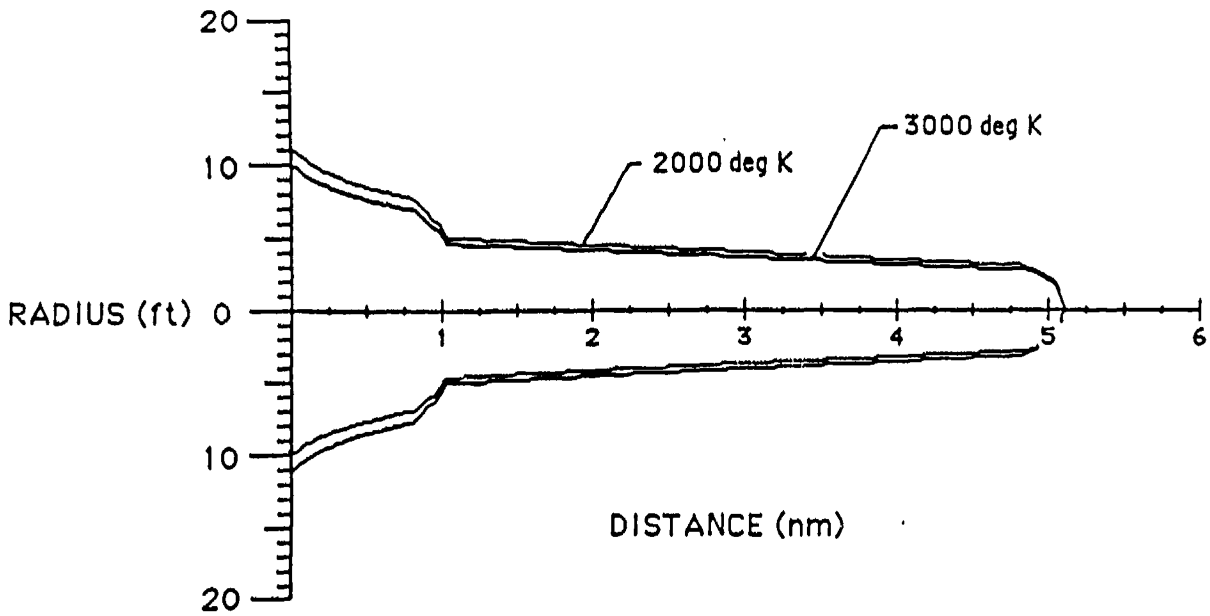


Initial Plume Velocity	Boost diameter	Sustain diameter
2000 meters/sec	12.94 ft	5.56 ft
3000 m/sec	17.99	8.21
4000 m/sec	22.70	10.67

Dual thrust missile with 3.2/13.4 sec burn times
Launch Velocity: .9M (555kts TAS)
Initial plume mean tmperature: 2000 deg K
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII for VARIOUS INITIAL PLUME CENTERLINE TEMPERATURES

20,000 ft launch altitude
20 sec elapse time
Level flight

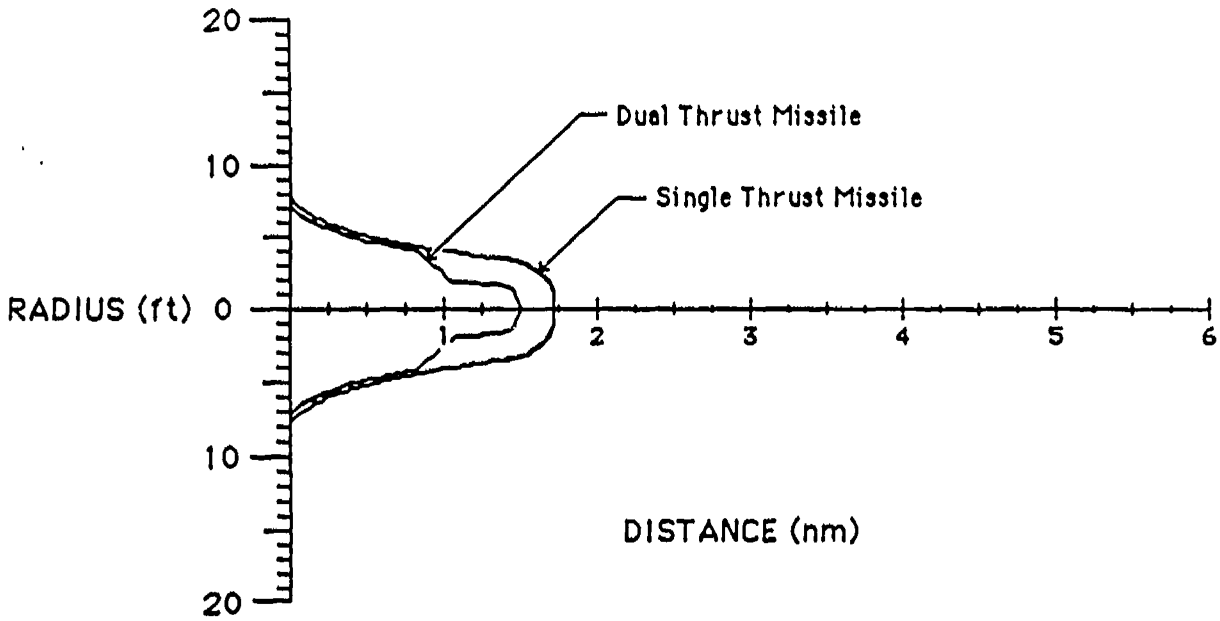


Initial Plume Temperature	Boost diameter	Sustain diameter
2000 degrees Kelvin	17.99 ft	8.21 ft
3000 deg K	16.18	7.48

Dual thrust missile with 3.2/13.4 sec burn times
Launch Velocity: .9M (555kts TAS)
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII COMPARISON BETWEEN "SINGLE" and "DUAL" THRUST MISSILES

20,000 ft launch altitude
.9M launch velocity
5 sec elapse time

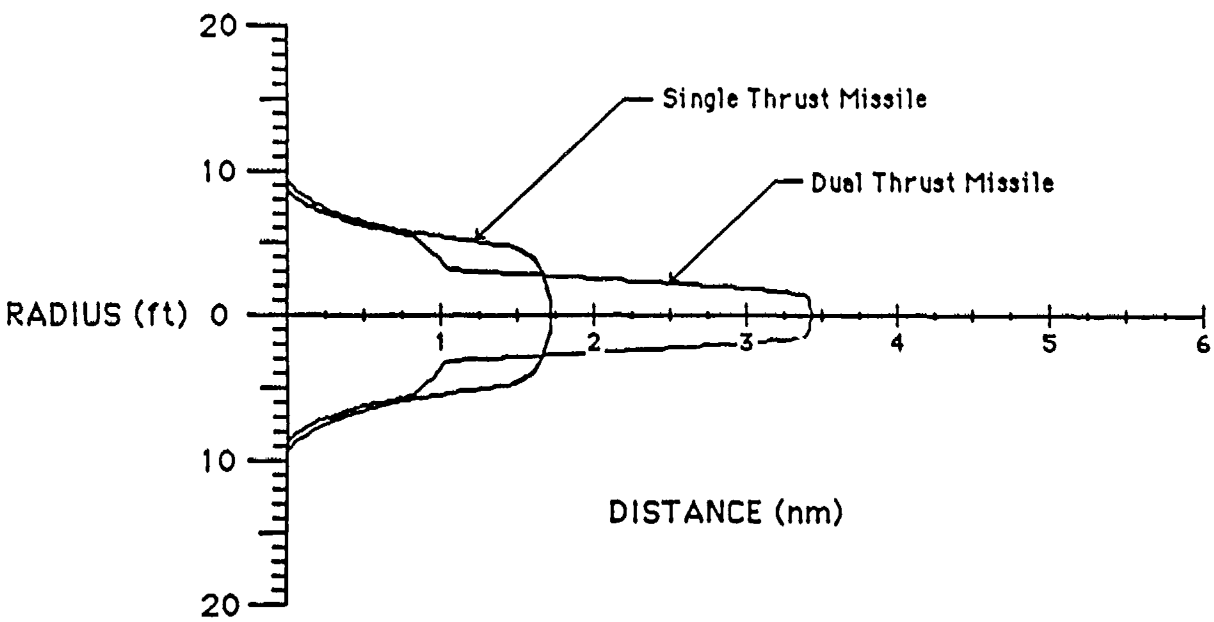


Missile	Burn Time(s)	Thrust	Plume Length
Single	4.4 sec	7500 lb	1.78 nm
Dual	3.4/13.4	6000/1000	1.49

Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII COMPARISON BETWEEN "SINGLE" and "DUAL" THRUST MISSILES

20,000 ft launch altitude
.9M launch velocity
10 sec elapse time

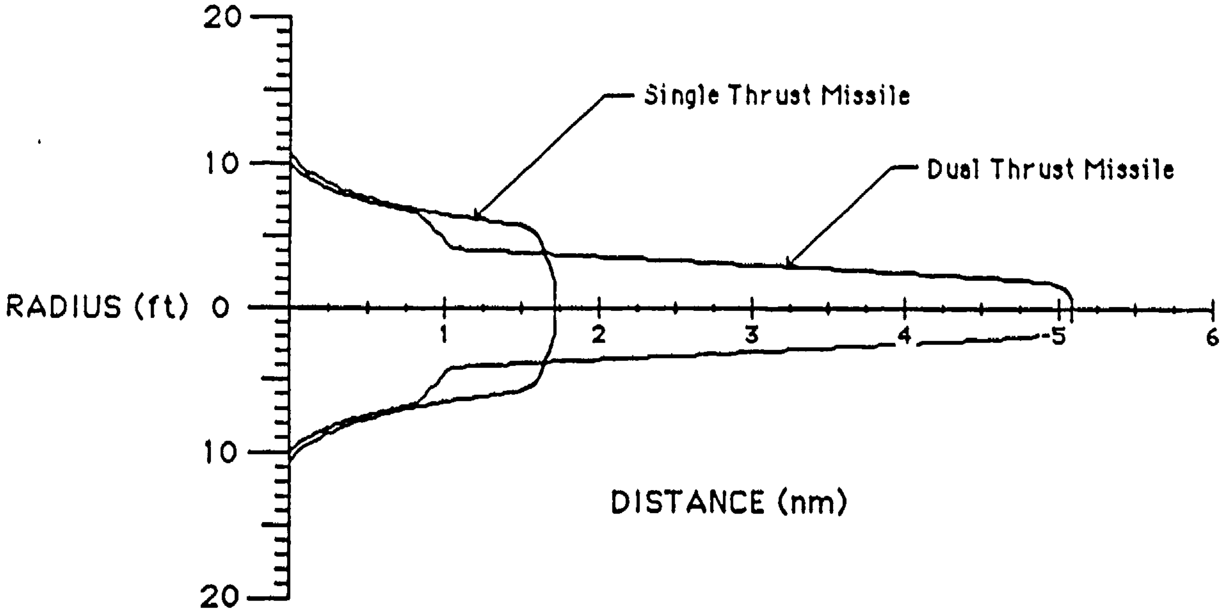


Missile	Burn Time(s)	Thrust	Plume Length
Single	4.4 sec	7500 lb	1.78 nm
Dual	3.4/13.4	6000/1000	3.47

Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

DOWN-RANGE PLUME RADII COMPARISON BETWEEN "SINGLE" and "DUAL" THRUST MISSILES

20,000 ft launch altitude
.9M launch velocity
15 sec elapse time



Missile	Burn Time(s)	Thrust	Plume Length
Single	4.4 sec	7500 lb	1.78 nm
Dual	3.4/13.4	6000/1000	5.09

Initial plume mean temperature: 2000 deg K
Initial plume mean velocity: 3000 m/sec
Initial missile weight: 500 lb

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